MODEL OF DARK MATTER AND DARK ENERGY BASED ON GRAVITATIONAL POLARIZATION

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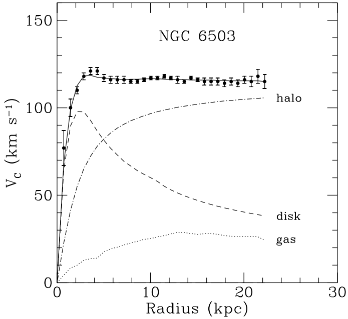
Rotation curves of galaxies are flat



For a circular orbit $v(r) = \sqrt{\frac{G\,M(r)}{r}}$

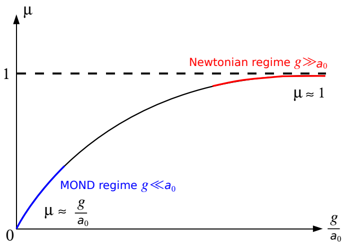
The fact that $v(r)$ is approximately constant implies that beyond the optical disk

$$M_{\rm halo}(r) \approx r \qquad \rho_{\rm halo}(r) \approx \frac{1}{r^2}$$

Modified Newtonian dynamics (MOND) [Milgrom 1983]

The Newtonian gravitational field is modified in an *ad hoc* way

$$\mu\left(\frac{g}{a_0}\right)\boldsymbol{g} = \boldsymbol{g}_{\text{Newtonian}} \quad \text{with} \quad a_0 \approx 1.2\,10^{-10}\,\text{m/s}^2$$



This value of a_0 is mysteriously close to the acceleration scale associated with the cosmological constant Λ

$$a_0 \approx 1.3\,a_\Lambda \quad \text{where} \quad a_\Lambda = \frac{1}{2\pi}\left(\frac{\Lambda}{3}\right)^{1/2}$$

Particle dark matter versus MOND

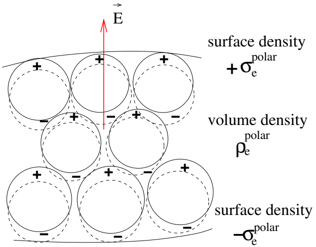
The alternative seems to be

1. Either accept the existence of cold dark matter, but
   - made of unknown non-baryonic particles yet to be discovered
   - which fails to reproduce in a natural way the flat rotation curves of galaxies
2. Or postulate a modification of the fundamental law of gravity: MOND and its relativistic extensions (TeVeS [Bekenstein 2004] and Einstein-æther), but
   - on an *ad hoc* and not yet physically justified basis

Here we shall adopt a different approach

1. Keep the standard law of gravity namely general relativity and its Newtonian limit when $c \to \infty$
2. Use the phenomenology of MOND to guess what could be the (probably unorthodox) nature of dark matter
3. Propose a natural physical mechanism for MOND
4. Develop a relativistic matter model and apply it to cosmology

The electric field in a dielectric medium



The atoms in a dielectric are modelled by electric dipole moments

$$\boldsymbol{\pi}_e = q\,\boldsymbol{\xi}$$

The polarization vector is

$$\boldsymbol{\Pi}_e = n\,\boldsymbol{\pi}_e$$

Density of polarization charges $\rho_e^{\text{polar}} = -\boldsymbol{\nabla}\cdot\boldsymbol{\Pi}_e$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = \frac{\rho_e + \rho_e^{\text{polar}}}{\varepsilon_0} \quad\Longleftrightarrow\quad \boldsymbol{\nabla}\cdot\left(\boldsymbol{E} + \frac{\boldsymbol{\Pi}_e}{\varepsilon_0}\right) = \frac{\rho_e}{\varepsilon_0}$$

The dipoles and polarization vector are aligned with the electric field

$$\frac{\boldsymbol{\Pi}_e}{\varepsilon_0} = \underbrace{\chi_e(E)}_{\text{electric susceptibility}}\ \boldsymbol{E}$$

Interpretation of the MOND equation [Blanchet 2006]

The MOND equation viewed as a modified Poisson equation [Bekenstein & Milgrom 1984]

$$\nabla \cdot \left[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \right] = -4\pi\, G\, \rho$$

is formally analogous to the equation of electrostatics inside a dielectric. We pose

$$\mu = 1 + \underbrace{\chi(g)}_{\substack{\text{gravitational}\\ \text{susceptibility}}} \quad \text{and} \quad \underbrace{\boldsymbol{\Pi}}_{\substack{\text{gravitational}\\ \text{polarization}}} = -\frac{\chi}{4\pi\, G}\, \boldsymbol{g}$$

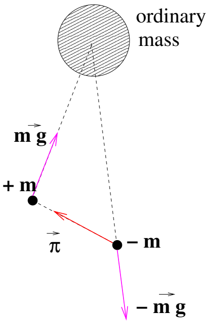
The MOND equation is equivalent to

$$\Delta U = -4\pi\, G\, (\rho + \rho_{\text{polar}})$$

In this interpretation the Newtonian law of gravity is not violated but we are postulating a new form of dark matter consisting of "polarization masses" with density

$$\boxed{\rho_{\text{polar}} = -\nabla \cdot \boldsymbol{\Pi}}$$

Sign of the gravitational susceptibility



The "digravitational" medium is modelled by individual dipole moments $\boldsymbol{\pi}$

$$\boldsymbol{\pi} = m\,\boldsymbol{\xi}$$
$$\boldsymbol{\Pi} = n\,\boldsymbol{\pi}$$

We suppose that the dipoles consist of particles doublets with

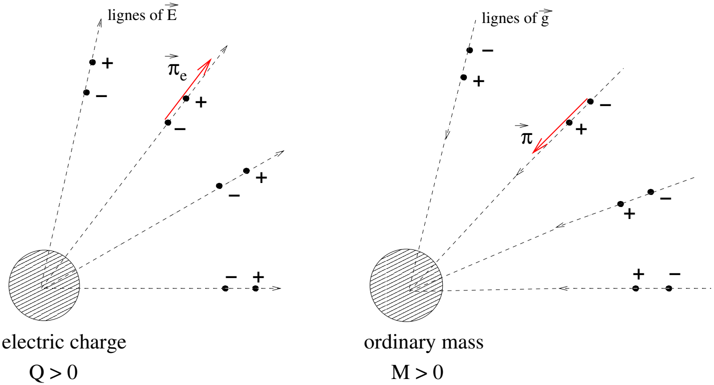
- opposite gravitational masses $m_\mathrm{g} = \pm m$
- positive inertial masses $m_\mathrm{i} = m$

1. The gravitational force is governed by a negative Coulomb law
2. The dipoles tend to align in the same direction as the gravitational field thus

$$\boxed{\chi < 0}$$

which is nicely compatible with MOND since $0 < \mu < 1 \Longrightarrow -1 < \chi < 0$

Electric screening versus gravitational anti-screening



Screening by polarization charges

$$\chi_e > 0$$

Anti-screening by polarization masses

$$\chi < 0$$

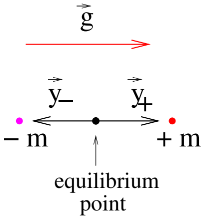
Interpretation of the dark matter medium

1. The dipole moments cannot be stable in a gravitational field and we must invoke some non-gravitational internal force (*i.e.* a "fifth force") which is attractive between unlike masses
2. The dark matter medium is like a gravitational plasma, *i.e.* a medium composed of particles
$$(m_\mathrm{i}, m_\mathrm{g}) = (m, \pm m)$$
in equal numbers (so the plasma is globally neutral) and with local number densities $n_\pm$
3. We find that to obtain MOND we must postulate the Gauss law

$$\nabla \cdot \boldsymbol{f}_\mathrm{int} = -\frac{4\pi\, G\, m}{\chi} \left(n_+ - n_-\right)$$

where $\chi < 0$ is the susceptibility of the dipolar medium

Gravitational plasma oscillations



Let $\boldsymbol{y}_\pm$ be the displacements from the equilibrium position at which the plasma is locally neutral, *i.e.* $n_+ = n_- = n$. The equation of motion is

$$m \frac{d^2 \boldsymbol{y}_\pm}{dt^2} = \pm m \left(\boldsymbol{f}_{\mathrm{int}} + \boldsymbol{g} \right)$$

Consider a small departure from equilibrium. The density perturbation reads $n_\pm = n \left(1 - \boldsymbol{\nabla} \cdot \boldsymbol{y}_\pm\right)$ to first order in $\boldsymbol{y}_\pm$

The dipole $\boldsymbol{\xi} = \boldsymbol{y}_+ - \boldsymbol{y}_-$ obeys the harmonic oscillator

$$\frac{d^2 \boldsymbol{\xi}}{dt^2} + \omega^2 \boldsymbol{\xi} = 2\boldsymbol{g}$$

in which ω is the usual plasma frequency given here by

$$\omega = \sqrt{-\frac{8\pi\, G\, m\, n}{\chi}}$$

Non viability of the quasi-Newtonian model

The quasi-Newtonian model

- Suggests that the gravitational analogue of the electric polarization is possible
- Yields a simple and natural explanation of the MOND equation
- Requires the existence of a new non-gravitational force
- Interprets the dark matter medium as a polarizable gravitational plasma

BUT THIS MODEL IS NOT VIABLE

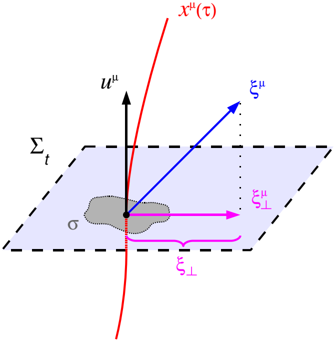
- Is not relativistic
- Involves negative gravitational masses so violates the equivalence principle
- Does not allow to answer questions related to cosmology

Action principle for dipolar fluid [Blanchet 2007]

We propose a matter action in standard general relativity of the type

$$S = \int d^4x \sqrt{-g}\, L\left[J^\mu, \xi^\mu, \dot{\xi}^\mu, g_{\mu\nu}\right]$$

where the current density J^μ and the dipole moment ξ^μ are two independent dynamical variables



- The current density $J^\mu = \sigma u^\mu$ is conserved

$$\nabla_\mu J^\mu = 0$$

- The covariant time derivative is denoted

$$\dot{\xi}^\mu \equiv \frac{\mathrm{D}\xi^\mu}{\mathrm{d}\tau} = u^\nu \nabla_\nu \xi^\mu$$

- Projection perpendicular to the velocity

$$\xi^\mu_\perp \equiv \perp_{\mu\nu} \xi^\nu$$

Dipolar dark matter Lagrangian [Blanchet & Le Tiec 2008]

$$L = \sigma \left[-1 - \sqrt{\left(u_\mu - \dot{\xi}_\mu\right)\left(u^\mu - \dot{\xi}^\mu\right)} + \frac{1}{2}\dot{\xi}_\mu \, \dot{\xi}^\mu \right] - \mathcal{W}(\Pi_\perp)$$

1. The mass term σ represents the inertial mass density of the dipole moments (say $\sigma = 2m\,n$)
2. The second term is a potential term inspired by the action of particles with spins in general relativity [Papapetrou 1951, Bailey & Israel 1980]
3. The third term is a kinetic-like term for the dipole moment and will tell how its evolution differs from parallel transport
4. The potential function $\mathcal{W}$ depends on the polarization field

$$\boxed{\Pi_\perp = \sigma\,\xi_\perp}$$

and describes a non-gravitational force $\mathcal{F}^\mu$ internal to the dipole moment

Equations of motion and evolution

Varying the action with respect to the dynamical variables J^μ and ξ^μ we find that one equation can be solved with $\left[\left(u_\mu - \dot{\xi}_\mu\right)\left(u^\mu - \dot{\xi}^\mu\right)\right]^{1/2} = 1$

Equation of motion of dipolar fluid

$$\underbrace{\dot{u}^\mu = -\mathcal{F}^\mu}_{\text{non-geodesic motion}} \qquad \text{where} \qquad \underbrace{\mathcal{F}^\mu \equiv \hat{\xi}^\mu_\perp \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\Pi_\perp}}_{\text{dipolar internal force}}$$

Equation of evolution of dipole moment

$$\dot{\Omega}^\mu = \frac{1}{\sigma}\nabla^\mu\left(\mathcal{W} - \Pi_\perp \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\Pi_\perp}\right) - \underbrace{\xi^\nu_\perp R^\mu_{\ \rho\nu\sigma} u^\rho u^\sigma}_{\text{coupling to Riemann curvature}}$$

$$\text{where} \quad \Omega^\mu \equiv \perp^\mu_\nu \dot{\xi}^\nu_\perp + u^\mu\left(1 + \xi_\perp \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\Pi_\perp}\right)$$

The equations depend only on the (space-like) perpendicular projection $\xi^\mu_\perp = \perp^\mu_\nu \xi^\nu$ which represents the physical dipole moment variable

Stress-energy tensor

Varying the action with respect to the metric we obtain

$$T^{\mu\nu} = \underbrace{r}_{\text{energy density}} u^\mu u^\nu + \underbrace{\mathcal{P}}_{\text{pressure}} \perp^{\mu\nu} + 2 \underbrace{Q^{(\mu}}_{\text{heat flux}} u^{\nu)} + \underbrace{\Sigma^{\mu\nu}}_{\text{anisotropic stresses}}$$

where

$$\begin{aligned}
r &= \rho + \mathcal{W} - \Pi_\perp \mathcal{W}' \\
\mathcal{P} &= -\mathcal{W} + \frac{2}{3} \Pi_\perp \mathcal{W}' \\
Q^\mu &= \sigma\, \dot{\xi}^\mu_\perp + \Pi_\perp \mathcal{W}' u^\mu - \Pi^\lambda_\perp \nabla_\lambda u^\mu \\
\Sigma^{\mu\nu} &= \left(\frac{1}{3} \perp^{\mu\nu} - \hat{\xi}^\mu_\perp \hat{\xi}^\nu_\perp\right) \Pi_\perp \mathcal{W}'
\end{aligned}$$

The mass density ρ contains the rest mass σ and a dipolar term $-\nabla_\lambda \Pi^\lambda_\perp$ which appears as a relativistic generalisation of the polarization mass density

$$\boxed{\rho = \sigma - \nabla_\lambda \Pi^\lambda_\perp}$$

Cosmological perturbations at large scales

1. Apply first-order perturbation theory around FLRW background

$$\begin{aligned} g_{\mu\nu} &= \overline{g}_{\mu\nu} + \delta g_{\mu\nu} \\ u^\mu &= \overline{u}^\mu + \delta u^\mu \\ \xi_\perp^\mu &= 0 + \delta\xi_\perp^\mu \quad \Longleftarrow \text{ dipole moment is perturbative} \\ \Pi_\perp &= 0 + \delta\Pi_\perp \quad \Longleftarrow \text{ polarization field is perturbative} \end{aligned}$$

2. Suppose that the potential takes the form

$$\mathcal{W}(\Pi_\perp) = \underbrace{\mathcal{W}_0}_{\text{cosmological constant}} + \frac{1}{2}\mathcal{W}_2\,\Pi_\perp^2 + \mathcal{O}\left(\Pi_\perp^3\right)$$

3. Apply standard SVT gauge-invariant formalism with $\delta\xi_\perp^\mu = \left(0, D^i y + y^i\right)$

- equations of motion

$$\begin{aligned} V' + \mathcal{H}\,V + \Phi &= -\mathcal{W}_2\,\overline{\sigma}\,a^2\,y \\ V_i' + \mathcal{H}\,V_i &= -\mathcal{W}_2\,\overline{\sigma}\,a^2\,y_i \end{aligned}$$

- evolution equations

$$\begin{aligned} y'' + \mathcal{H}\,y' - \mathcal{W}_2\,\overline{\sigma}\,a^2\,y &= 0 \\ y_i'' + \mathcal{H}\,y_i' - \mathcal{W}_2\,\overline{\sigma}\,a^2\,y_i &= 0 \end{aligned}$$

Agreement with the Λ-CDM scenario

1. To first order in the perturbation, $T^{\mu\nu} = T^{\mu\nu}_{\text{de}} + T^{\mu\nu}_{\text{dm}}$ where

$$\begin{aligned} T^{\mu\nu}_{\text{de}} &= -\mathcal{W}_0\, g^{\mu\nu} \\ T^{\mu\nu}_{\text{dm}} &= \rho\, u^\mu\, u^\nu + \underbrace{2Q^{(\mu}\, u^{\nu)}}_{\text{heat flux term}} \end{aligned}$$

2. Posing $\widetilde{u}^\mu = u^\mu + Q^\mu/\overline{\rho}$ we can recast the dark matter fluid into a perturbed pressureless perfect fluid at linear perturbation order

$$\begin{aligned} T^{\mu\nu}_{\text{dm}} &= \rho\, \widetilde{u}^\mu\, \widetilde{u}^\nu \\ \text{where}\quad \rho &= \underbrace{\sigma}_{\substack{\text{rest mass}\\ \text{energy density}}} - \underbrace{D_i \Pi^i_\perp}_{\substack{\text{dipolar polarization}\\ \text{energy density}}} \end{aligned}$$

The dipolar fluid is undistinguishable from

- standard DE (a cosmological constant $\Lambda = 8\pi\mathcal{W}_0$)
- standard CDM (a pressureless perfect fluid)

at the level of first-order cosmological perturbations

Non-relativistic limit of the model ($c \to +\infty$)

1. Equation of motion of the dipolar particle

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \underbrace{\boldsymbol{g}}_{\text{local gravitational field}} - \underbrace{\boldsymbol{\mathcal{F}}}_{\text{internal force}}$$

2. Evolution equation of the dipole moment

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} = \boldsymbol{\mathcal{F}} + \frac{1}{\sigma}\boldsymbol{\nabla}\left(\mathcal{W} - \Pi\,\mathcal{W}'\right) + \underbrace{(\boldsymbol{\xi}\cdot\boldsymbol{\nabla})\,\boldsymbol{g}}_{\text{tidal effect}}$$

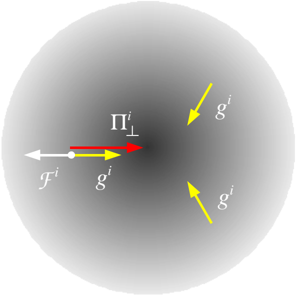
3. Conservation of the number of particles

$$\partial_t \sigma = -\boldsymbol{\nabla}\cdot(\sigma\boldsymbol{v})$$

4. Poisson equation for the gravitational potential

$$\boldsymbol{\nabla}\cdot\boldsymbol{g} = -4\pi\Big(\underbrace{\rho_{\mathrm{b}}}_{\text{baryonic matter}} + \sigma - \boldsymbol{\nabla}\cdot\boldsymbol{\Pi}\Big)$$

Dipolar dark matter in a central gravitational field



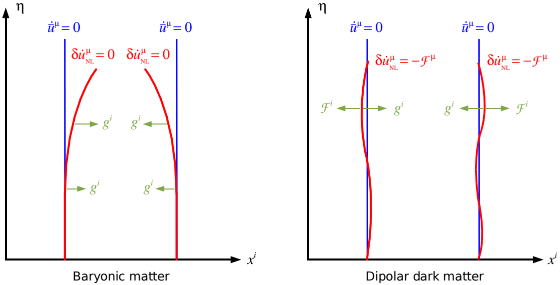
In spherical symmetry we find that there is a solution for which the dipolar fluid is static (i.e. at rest), the dipole moment is aligned with the gravitational field, and varies on a very long time-scale so that it is practically in equilibrium

$$\begin{aligned} \boldsymbol{v} &= \mathbf{0} \\ \boldsymbol{\Pi} &= -\Pi(r)\,\boldsymbol{e}_r \end{aligned}$$

In this solution the internal force balances the gravitational field

$$\boldsymbol{\mathcal{F}} = \boldsymbol{g}$$

Weak clustering of dipolar dark matter



- Baryonic matter follows the geodesic equation $\dot{u}^\mu = 0$, therefore collapses in regions of overdensity
- Dipolar dark matter obeys $\dot{u}^\mu = -\mathcal{F}^\mu$, with the internal force $\mathcal{F}^i$ balancing the gravitational field g^i created by an overdensity

We expect that the mass density of dipolar dark matter in a galaxy at low redshift remains close to its mean cosmological value

$$\sigma \approx \overline{\sigma} \ll \rho_\mathsf{b} \quad \text{and} \quad \boldsymbol{v} \approx \mathbf{0}$$

Recovering MOND in a galaxy at low red-shift

Using $\boldsymbol{v} \approx \mathbf{0}$ in the equation of motion

$$\boldsymbol{g} = \hat{\mathbf{\Pi}} \frac{d\mathcal{W}}{d\Pi} \quad \Longleftarrow \quad \text{polarization of the dipolar medium}$$

Using $\sigma \approx \overline{\sigma} \ll \rho_{\mathsf{b}}$ in the gravitational field equation

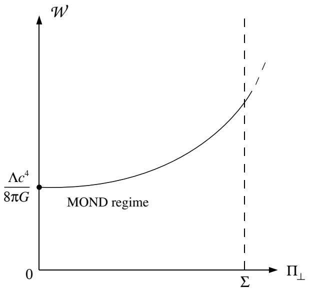
$$\boldsymbol{\nabla} \cdot \Big[\boldsymbol{g} - 4\pi\, \mathbf{\Pi}\Big] = -4\pi\, \rho_{\mathsf{b}} \quad \Longleftarrow \quad \text{modification of the Poisson equation}$$

1. Since the polarization $\mathbf{\Pi}$ is aligned with the gravitational field $\boldsymbol{g}$ we recover the MOND equation with MOND function $\mu = 1 + \chi$ where

$$\mathbf{\Pi} = -\frac{\chi}{4\pi}\, \boldsymbol{g}$$

2. There is one-to-one correspondence between μ and the potential $\mathcal{W}$

The fundamental potential $\mathcal{W}$



The potential $\mathcal{W}$ is determined through third order

$$\mathcal{W} = \frac{\Lambda}{8\pi} + 2\pi\,\Pi_\perp^2 + \underbrace{\frac{16\pi^2}{3a_0}\,\Pi_\perp^3}_{\text{MOND acceleration appears here}} + \mathcal{O}\left(\Pi_\perp^4\right)$$

Though the model is purely *classical* it is tempting to interpret the cosmological constant Λ as a "vacuum polarization", i.e. the residual polarization that remains when the "classical" part of the polarization $\Pi_\perp \to 0$

Order of magnitude of the cosmological constant

1. Introduce a purely numerical coefficient α such that

$$a_0 = \frac{1}{2\pi\,\alpha}\left(\frac{\Lambda}{3}\right)^{1/2} \qquad \left(\text{i.e.}\quad a_0 = \frac{a_\Lambda}{\alpha}\right)$$

2. Write the fundamental potential as $\mathcal{W} = \frac{3\pi\, a_0^2}{2}\, f\left(\frac{\Pi_\perp}{a_0}\right)$ with

$$f(x) = \underbrace{\alpha^2 + \frac{4}{3}\,x^2 + \frac{32\pi}{9}\,x^3 + \mathcal{O}\left(x^4\right)}_{\text{some ``universal'' function of } x \,\equiv\, \Pi_\perp / a_0}$$

3. The numerical coefficients in $f(x)$ are expected to be of the order of one, hence the cosmological constant should be of the order of

$$\boxed{\Lambda \sim a_0^2}$$

in good agreement with the observations (which give $\alpha \approx 0.8$)

Conclusions

This model based on gravitational polarization:

1. Offers a natural physical explanation of the phenomenology of MOND
2. Benefits from both the successes of MOND at galactic scales and of Λ-CDM at cosmological scales
3. Yields a nice unification between the dark energy in the form of Λ and the dark matter *à la* MOND
4. But is "phenomenological" and presumably only valid in a regime of weak gravitational fields

More work should be done to:

1. Connect the model to fundamental (quantum) physics valid at microscopic scale [work in progress]
2. Investigate second-order perturbations in cosmology
3. Numerically compute the non-linear growth of perturbations and compare with large-scale structures
4. Test the model at the intermediate scale of galaxy clusters