Regular DGP in the core of a hypermonopole

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with Christophe Ringeval

UCL

Introduction

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- Static ansatz

$$ds^2 = e^{\sigma(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + \omega(r)^2 d\Omega^2$$

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- Gauge fields

$$\vec{C}_\theta = \frac{1-Q(r)}{q}\vec{u}_\phi \quad \vec{C}_\phi = -\frac{1-Q(r)}{q}\sin\theta\vec{u}_\theta$$

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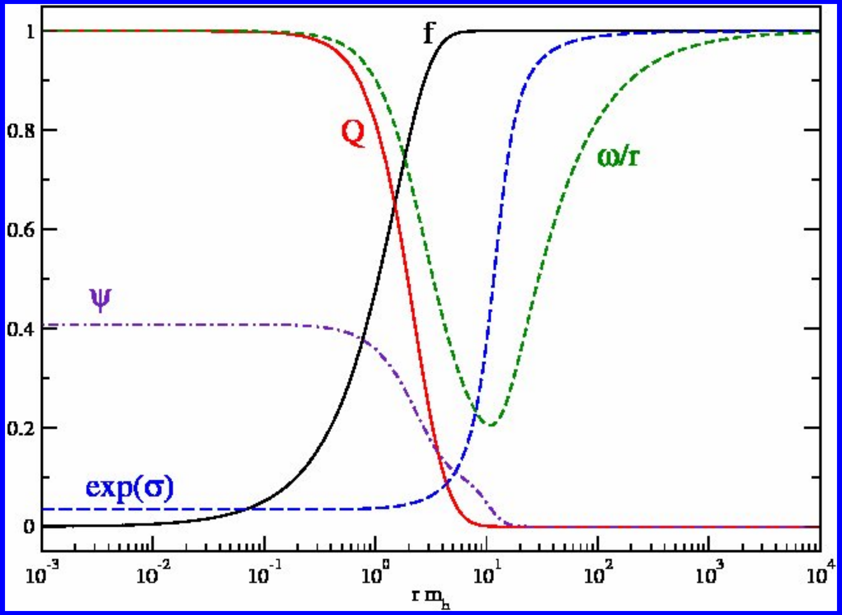
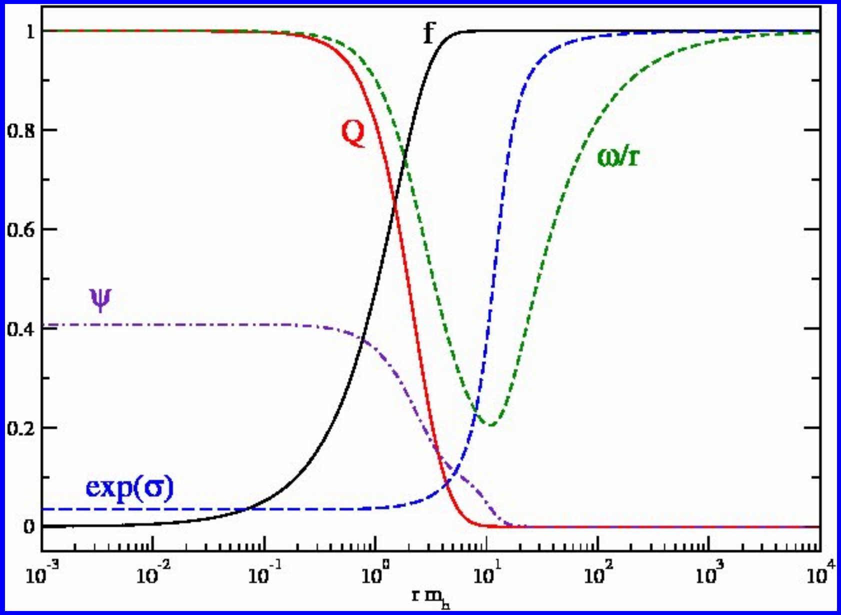
- Define

$$\alpha \equiv \kappa^2 v^2 \quad \epsilon \equiv \frac{q^2 v^2}{\lambda v^2} \quad \beta \equiv \frac{m_d^2}{\lambda v^2}$$



Example $\alpha = 2$, $\epsilon = 0.5$, $\beta = 1$. Typical topological configuration for the fields. Minkowski at boundary. TWPBVP: Cash and Mazzia algorithm, JCAM (2005).



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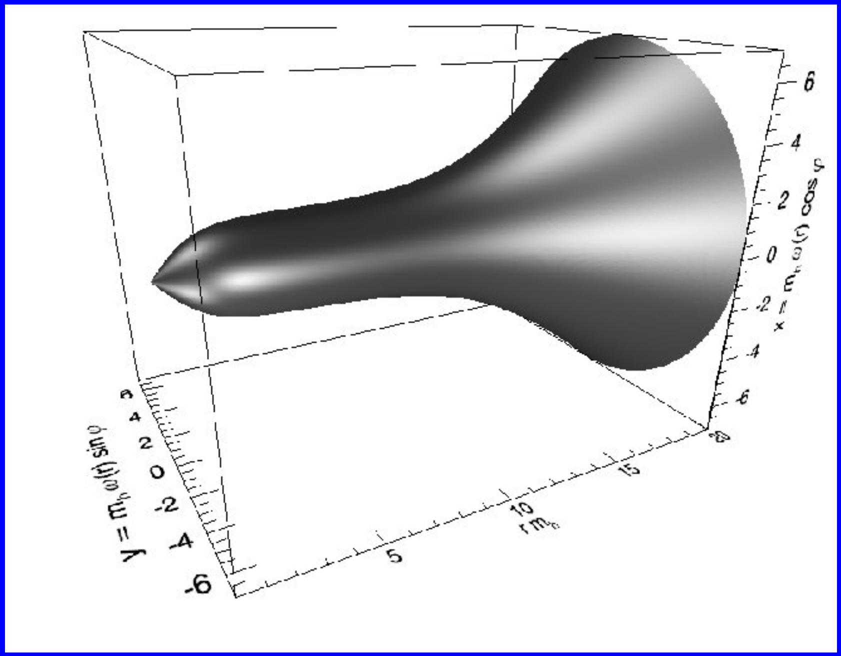

Region where ω no longer grows, cylindrically shaped ED, resonant gravitons at this scale

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- Monopole: no singularities

Tensor fluctuations

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- FT+SH: $\Box \to M^2$ and $L^2 \to -l(l+1)$, $\xi(0) \to 0$.

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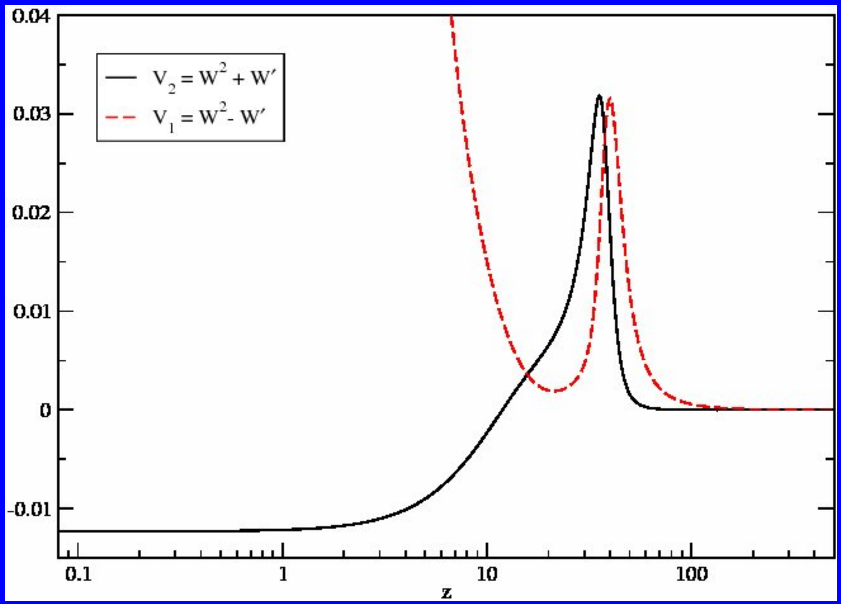

Depends on α, β, ϵ. Fro V_2: ground state ξ_0 not normalizable. For V_1, $1/\xi_0$ not regular in 0. SUSY is broken and spectrum $M^2 > 0$.

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- Using retarded Green function, for TT source $S_{\mu\nu}$ [Garriga, Tanaka PRL (2000)]

$$h_{\mu\nu}(X_1) = -\frac{2\kappa^2}{m_h^2\omega}e^{-\psi(z_1)-3\sigma/4}$$
$$\times \int G_\xi(X_1;X_2)e^{-\psi(z_2)+3\sigma/4}\omega(z_2)S_{\mu\nu}d^7X_2$$

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- Solution

$$h^{\flat}_{\mu\nu} = \lim_{z\to 0}\frac{\kappa^2}{8\pi^2 m_h^2}\int d^3\vec{x}_2 s_{\mu\nu}\int dM \frac{|u^{\flat}_{M,0}(z)|^2}{z^2}\frac{e^{-M\Delta\vec{x}}}{|\Delta\vec{x}|}$$

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- $4\pi^3$: $d-2$ times the surface of $d-1$ unit sphere.

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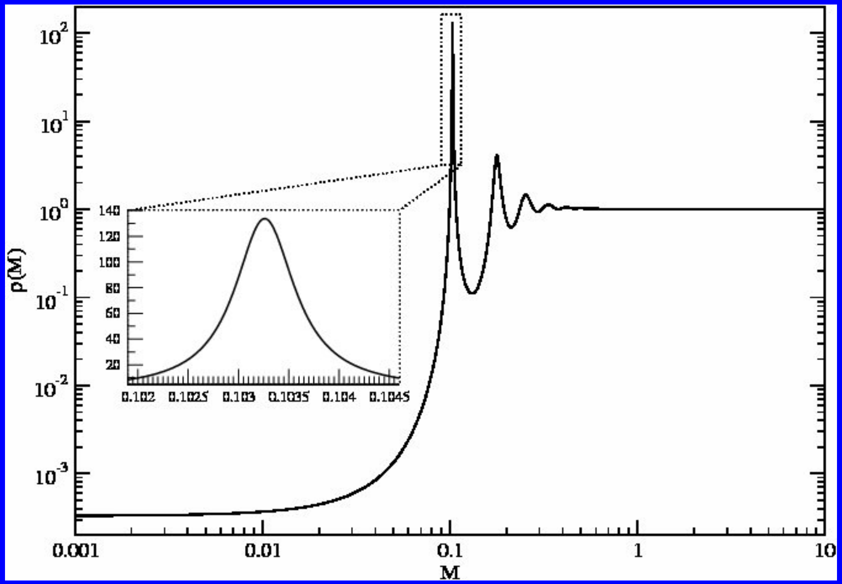
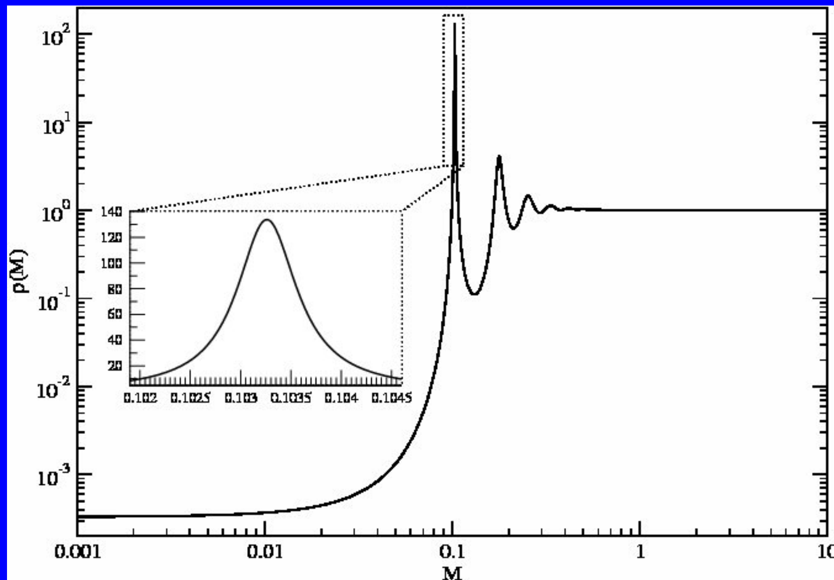
- Therefore one finds

$$h_{\mu\nu} = \frac{2\kappa^2}{8\pi^3 m_h^2} \int d^3\vec{x}_2 \frac{\mathcal{L}\{\rho(M) M^2\}}{|\Delta\vec{x}|} s_{\mu\nu}(\vec{x}_2)$$



There are at least two trapped metastable gravitons. Mass $m_g \approx 0.103$ Breit-Wigner with $\Gamma \approx 3.5 \times 10^{-4}$ (m_h units)



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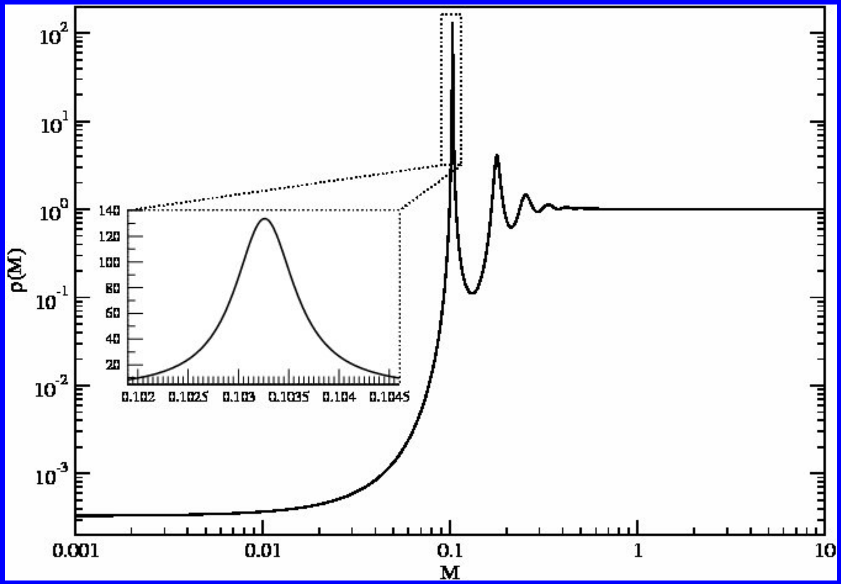

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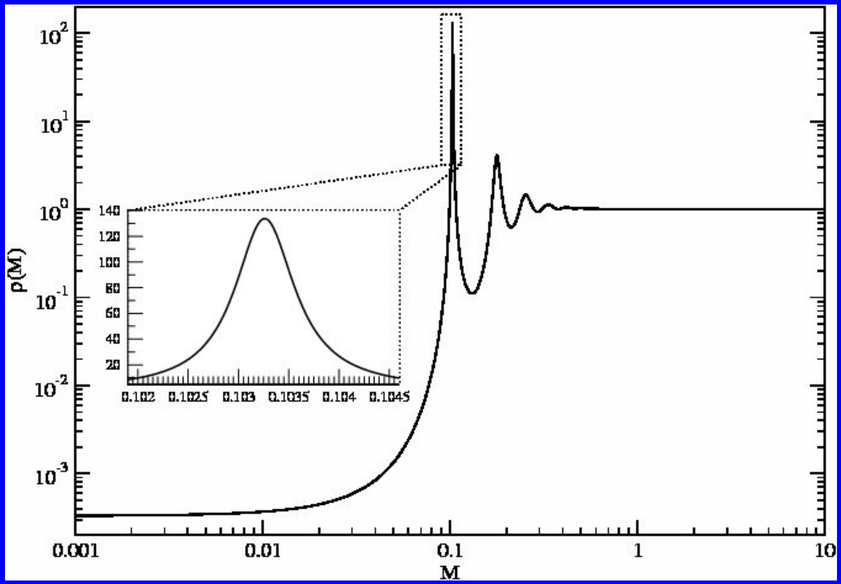

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- 7D at small/large distance
- We observed fractional power

Green function

$$G''_\xi + (-V_2 + \Box + e^\sigma \omega^{-2} L) G_\xi = \delta^4(x^\mu - x^{\mu\prime})$$
$$\times\, \delta(\cos\theta - \cos\theta')\delta(\phi - \phi')\delta(z - z')$$

$$G_\xi = -\int \frac{d^4p}{(2\pi)^4} e^{ip_\mu(x_1^\mu - x_2^\mu)} \sum_{l,m} Y_l^m(\theta_1, \phi_1) Y_l^{m*}(\theta_2, \phi_2)$$
$$\times \int \frac{u_{M,l}(z_1) u^*_{M,l}(z_2) dM}{M^2 + \vec{p}^{\,2} - (p^0 + i\epsilon)^2}$$