Unique F(R) Theory from the Standard Model

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We all know “the problem”

$G_{\mu\nu} \neq 8\pi G\,(T_{\mu\nu})_{\text{known}}$ on galaxy scales & larger

We also know the possible solutions:

1. Invent more $T_{\mu\nu}$
   - $\rho_{\rm DM} \sim 6\times\rho_{\rm known}$, $\rho_{\rm DE} \sim 18\times\rho_{\rm known}$
   - This works . . . but it’s epicyclic
2. Change gravity
   - $R \Rightarrow F(R)$ is the only stable way
   - This works . . . but it’s ALSO epicyclic
   - UNLESS you can DERIVE F(R) from 1st Principles

V_{eff} in de Sitter
$ds^2 = -dt^2 + a^2\, dx^i\, dx^i$, $a = e^{Ht}$

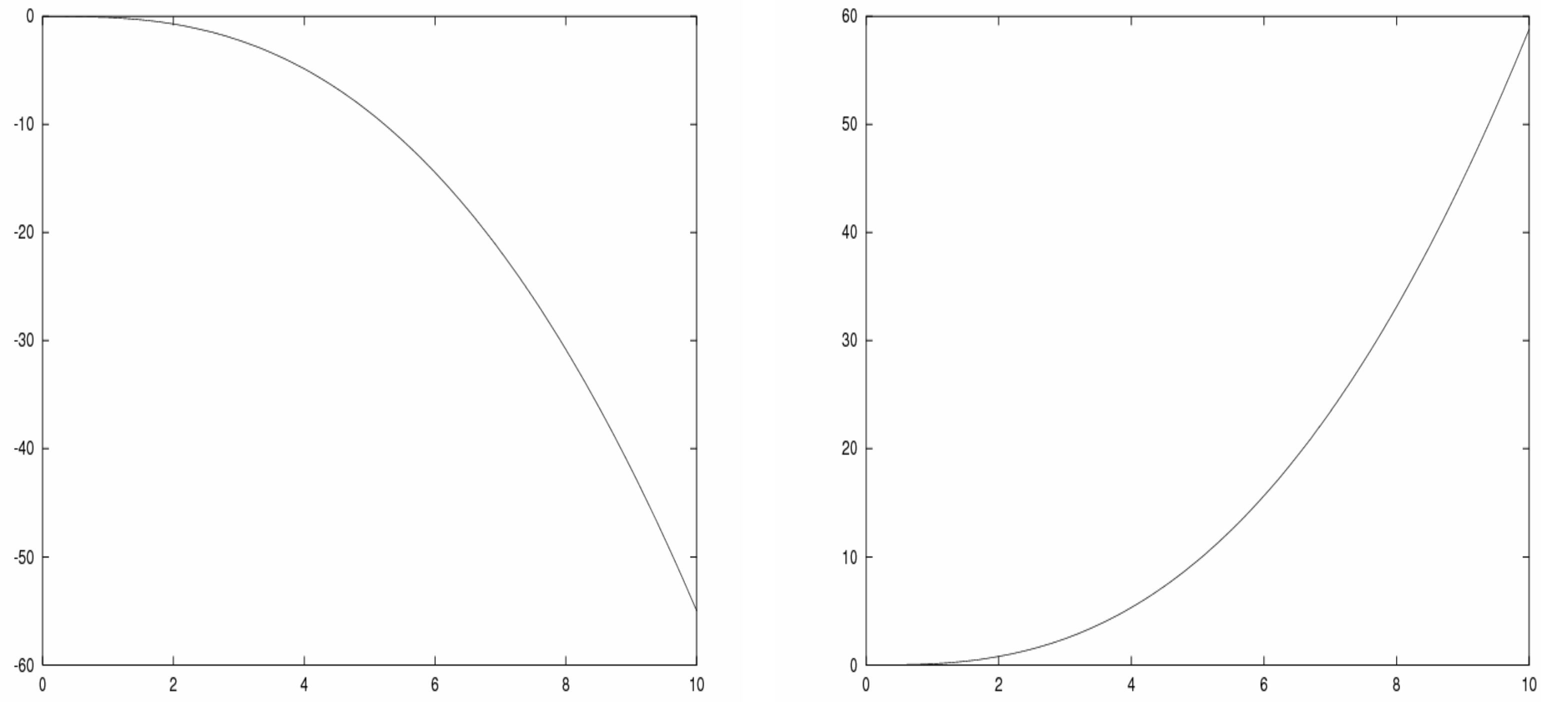
Yukawa: $\Delta\mathcal{L} = -f\varphi\overline{\psi}\psi\sqrt{-g}$

- $V_{\text{eff}} \equiv -\frac{H^4}{8\pi^2} \times F(z)$ with $z \equiv \frac{f^2\varphi^2}{H^2}$
- $F(z) = 2\gamma z - [\zeta(3) - \gamma]z^2$
$$+ \int_0^z dx(1+x)\Big[\psi(1+i\sqrt{x}) + \psi(1-i\sqrt{x})\Big]$$
$$\longrightarrow \frac{1}{2}z^2 \ln(z) + O(z^2)$$

SQED: $\Delta\mathcal{L} = ieA_\mu\varphi^*\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} + \ldots$

- $V_{\text{eff}} \equiv +\frac{3H^4}{8\pi^2} \times F(z)$ with $z \equiv \frac{e^2\varphi^*\varphi}{H^2}$
- $F(z) = [2\gamma - 1]z - [\frac{3}{2} - \gamma]z^2$
$$+ \int_0^z dx(1+x)\Big[\psi(\frac{3}{2} + \frac{1}{2}\sqrt{1-8x}) + \psi(\frac{3}{2} - \frac{1}{2}\sqrt{1-8x})\Big]$$
$$\longrightarrow \frac{1}{2}z^2 \ln(z) + O(z^2)$$

V_{eff} falls for Yukawa and grows for SQED



What is "H" for general $g_{\mu\nu}$?

Recall Yukawa

- $V_{eff} = -H^4/8\pi^2 \, F(f^2\varphi^2/H^2)$

In de Sitter

- $R^{\rho}{}_{\sigma\mu\nu} = H^2 \, (\delta^{\rho}{}_{\mu} g_{\sigma\nu} - \delta^{\rho}{}_{\nu} g_{\sigma\mu})$

Possibilities for H^2

- $H^2 \Rightarrow R/12$
- $H^2 \Rightarrow (R^{\mu\nu} R_{\mu\nu}/36)^{1/2}$
- $H^2 \Rightarrow (R^{\rho\sigma\mu\nu} R_{\rho\sigma\mu\nu}/24)^{1/2}$

Spacetime Exp. Strengthens QFT

WHY?

- Loops $\Rightarrow$ classical physics of virtuals
- Expansion $\Rightarrow$ holds virtuals apart longer

MAXIMUM EFFECT FOR:

- Inflation
- Massless and NOT conformally invariant

TWO PARTICLES:

- $m=0$ and $\xi=0$ Scalars
- Gravitons

Infrared Logarithms

WHAT: factors of ln(a) = Ht in QFT loops

FOR EXAMPLE: $\lambda\varphi^4$

- $p=(\lambda H^4/16\pi^2)[-2\ln^2(a) - 7/2\ln(a)] + O(\lambda^3)$

WHY:

- $i\Delta(x;x) = UV + (H^2/4\pi^2)\ln(a)$
- $\int_0^t dt'\, 1 = \ln(a)/H$

- NB: ln(a) even in <u>Power Spectrum</u>
  - Weinberg, hep-th/0605244

Leading Log Approximation

- General Expansion for $\Delta\mathcal{L} = -f\varphi\overline{\psi}\psi\sqrt{-g}$

$$\sum_n f^{2n}\Big\{\alpha_n[\ln(a)]^n + \beta_n[\ln(a)]^{n-1} + \ldots\ldots\Big\}$$

$$\Rightarrow \sum_n \alpha_n[f^2\ln(a)]^n$$

- Only $\varphi' s$ gives $\ln(a)$

$$\Rightarrow \quad \text{Integrate out } \psi' s \text{ and drop } \partial' s$$

- Starobinskiĭ gets α_n (not β_n etc.)

$$\Big\langle A(\varphi(t))\Big\rangle = \int dx A(x)\rho(t,x)$$

- Even late time limits

$$\rho(t,x) \quad \Rightarrow \quad N\exp\Big[-\frac{8\pi^2}{H^4}V_{eff}(x)\Big]$$

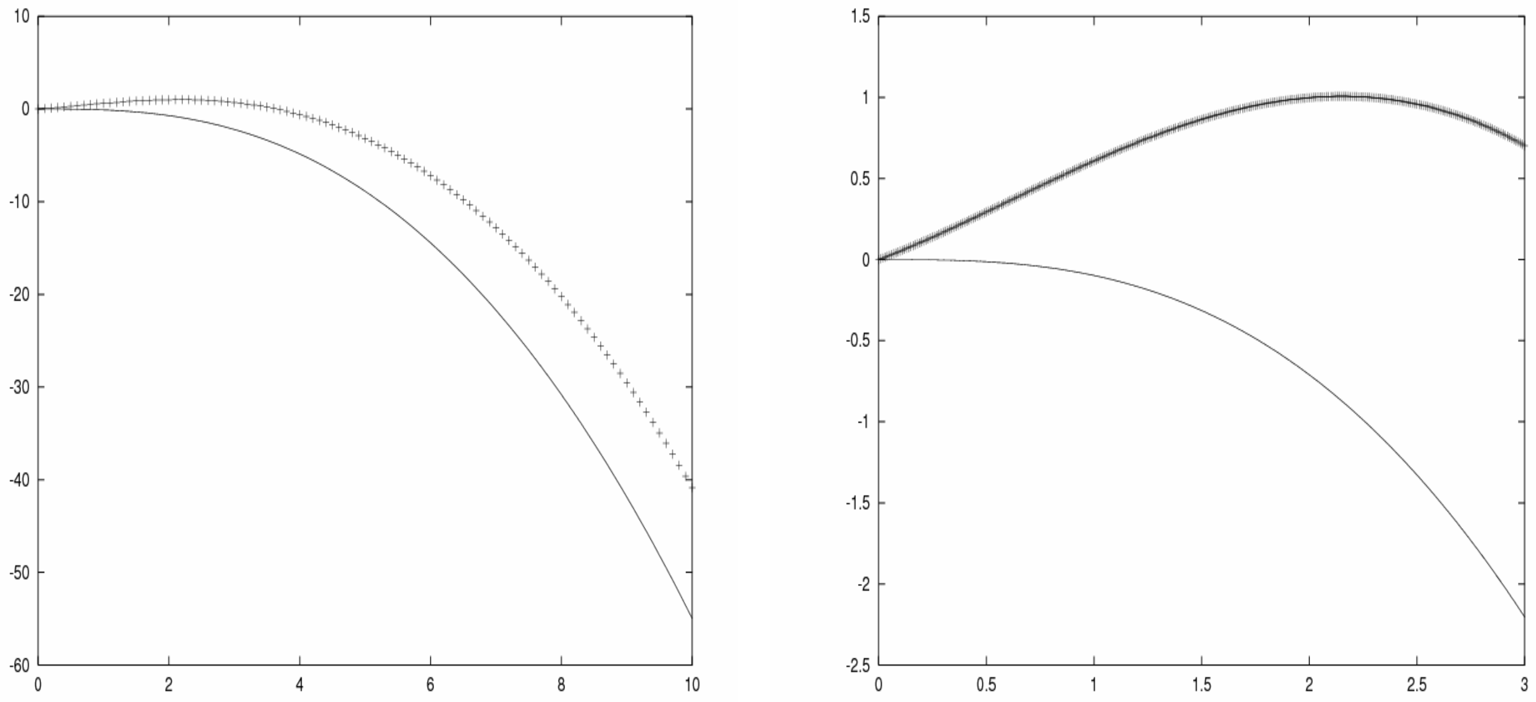
Stress Tensor at Leading Log

- Integrate out ψ's
- Drop derivatives
- $\langle T_{\mu\nu}\rangle = -g_{\mu\nu} \times V_s$
- Yukawa: $V_s = -H^4/8\pi^2 \, F_s(f^2\varphi^2/H^2)$

$$\begin{aligned} F_s(z) &= [\gamma - \frac{1}{2}]z - [\zeta(3) - \gamma + \frac{1}{4}]z^2 \\ &+ \frac{1}{2}(z + z^2)\Big[\psi(1 + i\sqrt{z}) + \psi(1 - i\sqrt{z})\Big] \end{aligned}$$

$$\begin{aligned} F(z) &= 2\gamma z - [\zeta(3) - \gamma]z^2 \\ &+ \int_0^z dx(1 + x)\Big[\psi(1 + i\sqrt{x}) + \psi(1 - i\sqrt{x})\Big] \end{aligned}$$

V_{eff} & V_s Similar for Large φ but Not for Small φ



$V_{eff} \neq V_s$ Fixes H^2

- True: $F_s(z) = \frac{1}{2} z F'(z) - \frac{1}{2} z - \frac{1}{4} z^2$
- For $\mathcal{L}_{eff} = -\Phi(R) (-g)^{1/2}$
  - $\langle T_{\mu\nu} \rangle \to -g_{\mu\nu} \{\Phi(R) - \frac{1}{2} R \Phi'(R)\}$
- For $V_{eff} = -H^4/8\pi^2 F(f^2\varphi^2/H^2)$

  $\to -(R/12)^2/8\pi^2 F(12f^2\varphi^2/R)$
- Would give $F_s(z) = \frac{1}{2} z F'(z)$
- The $-\frac{1}{2} z - \frac{1}{4} z^2$ from counterterms
  - $\delta\xi\varphi^2 R$: $\ln(H^2) \to \ln(\Lambda/3)$
  - $\delta\lambda\varphi^4$: $\ln(H^2) \to \ln(\Lambda/3)$

Summary

F(R) models can give a(t)

- but epicyclic
- <u>UNLESS</u> from fundamental theory

In de Sitter: $V_{eff} \sim H^4 F(\varphi^2/H^2)$

Leading Log Expansion fixes "H"

- $H^2 \Rightarrow R/12$ from propagator
- $\ln(H^2) \Rightarrow$ from counterterms

Next Step: Explore Cosmology