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# Dynamical models for dark matter halos with universal properties

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# Dark matter halos

*N*-body simulations have become the standard way to investigate the structure, dynamics and evolution of dark matter halos.

They revealed several "universal" properties:

- A "universal" density profile

$$\gamma(r) = -\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r}(r) = \frac{\gamma_0 + \gamma_\infty (r/r_s)^\eta}{1 + (r/r_s)^\eta}$$

- A power-law pseudo-phase space density

$$Q(r) = \rho/\sigma^3(r) \propto r^{-\alpha}$$

- A linear density slope – velocity dispersion relation

$$\beta(\gamma) \simeq 1 - 1.15(1 + \gamma/6)$$

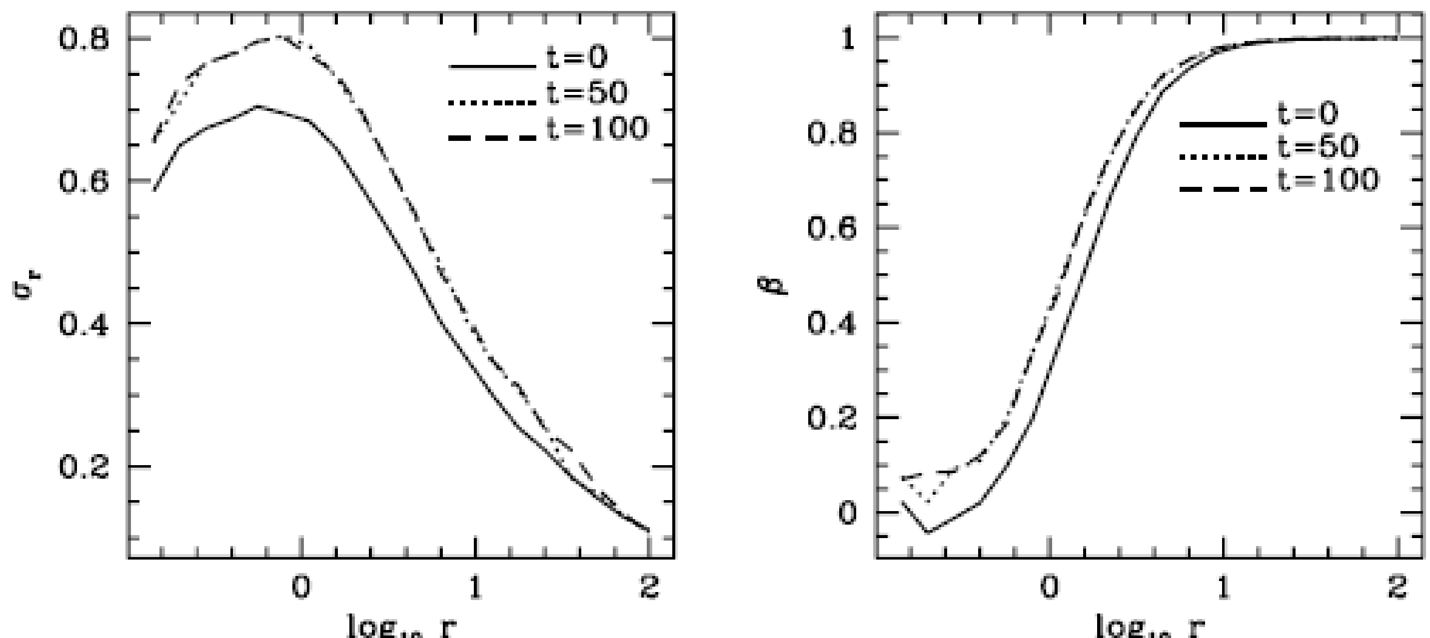
$$\beta(r) = 1 - \frac{\sigma_\theta^2(r)}{\sigma_r^2(r)}$$

Can we construct analytical dynamical models with these properties?

# Analytical dynamical models

Equilibrium halos described by a phase-space DF $F(\vec{r}, \vec{v})$

- "noise-free" description of the dynamical structure
  - simple toy models to represent a galaxy or dark matter halo
  - get a better insight into the phase-space structure
- Generate the initial conditions for realistic N-body models




Kazantzidis et al. 2004

# Constructing dynamical models

Dynamical model = phase-space distribution function $F(\vec{r},\vec{v})$

- contains all possible dynamical information
- can be written in terms of the integrals of motion
- yields direct view on the orbital structure

## spherical case

anisotropy $\beta(r)$

density $\rho(r)$

potential $\psi(r)$

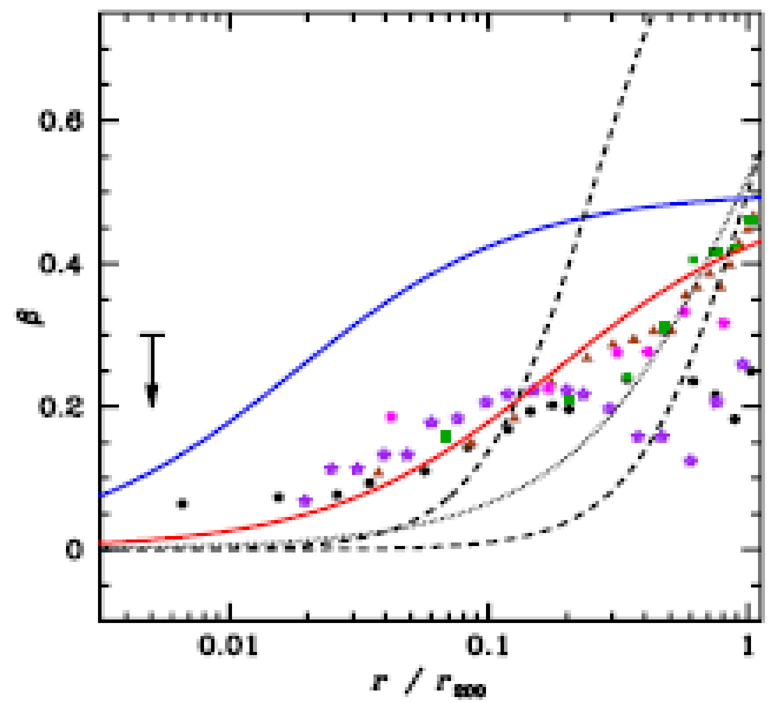
↓

a phase-space distribution function $F(\vec{r},\vec{v}) = F(E,L)$

# Constructing dynamical models

- mathematics are extremely complicated
  (non-linear integral transforms)
- only analytical solutions for a few sets of anisotropy constraints
  - systems with isotropy or a constant anisotropy
  - Osipkov-Merritt and Cuddeford models
    (completely radially anisotropic at large radii)

**These assumptions are not realistic**




Mamon & Lokas 2005

# More realistic models…

- how to build more realistic models ?
- augmented density formalism:
  one-to-one relation between $F(E,L)$ and $\tilde{\rho}(\psi,r)$
- problem: formulae complicated and numerically unstable…

$$\tilde{\rho}(\psi,r) = 2\pi M_{\mathrm{tot}} \int_0^{\psi} \mathrm{d}E \int_0^{2(\psi-E)} \frac{F(E, r v_T)}{\sqrt{2(\psi-E)-v_T^2}} \, \mathrm{d}v_T^2$$

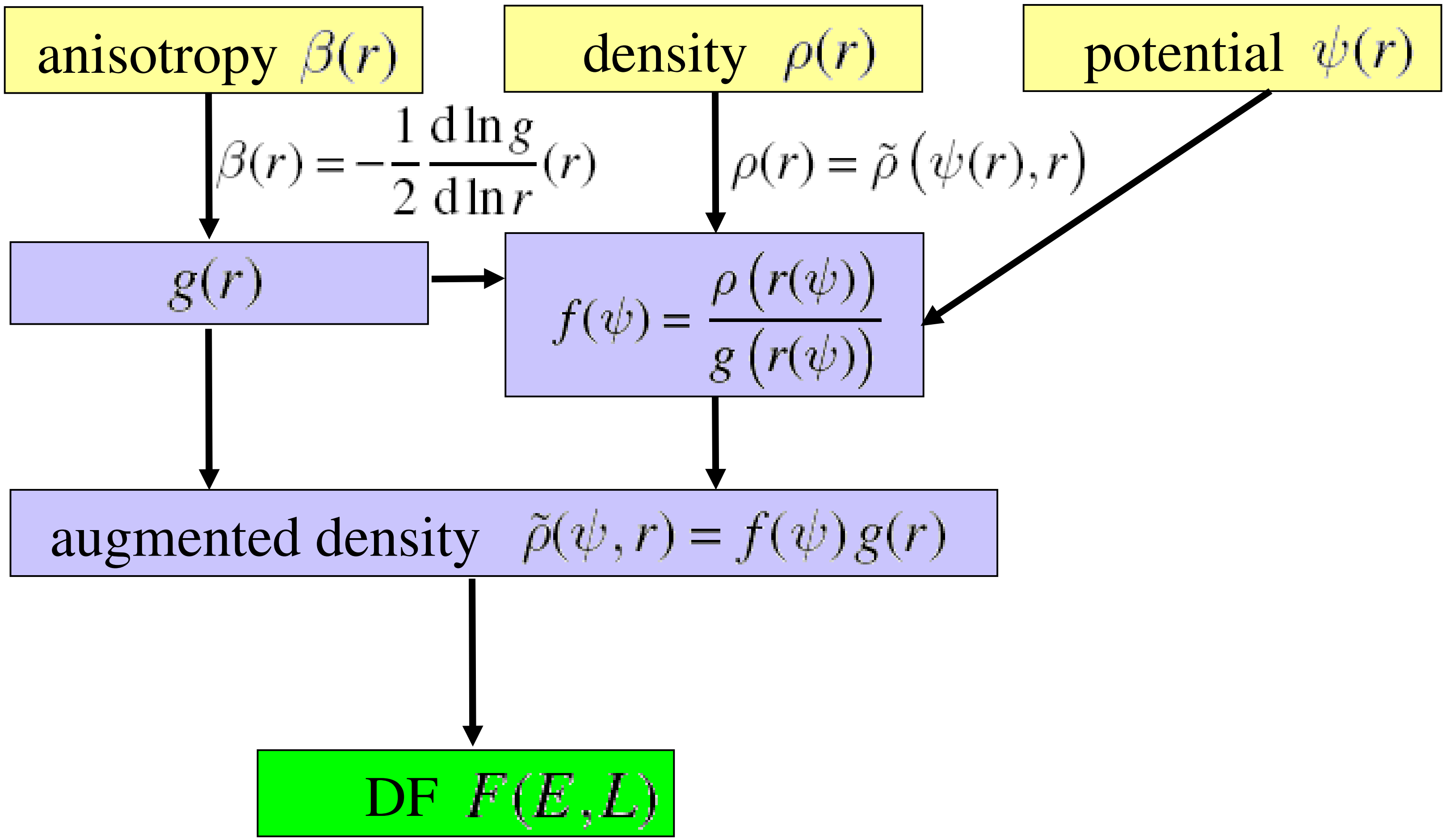
Dejonghe 1986

- things are easier for separable functions

$$\tilde{\rho}(\psi,r) = f(\psi)\, g(r)$$

the anisotropy is completely determined by $g(r)$

# More realistic models…



# Practical construction

derivation of the augmented density looks fine in theory….
practically impossible for general anisotropy profiles

⇒ solution: parameterized functions

$$\beta(r) = \frac{\beta_0 + \beta_\infty (r/r_{\rm a})^{2\delta}}{1+(r/r_{\rm a})^{2\delta}}$$

$$g(r) = \left(\frac{r}{r_{\rm a}}\right)^{-2\beta_0} \left(1+\frac{r^{2\delta}}{r_{\rm a}^{2\delta}}\right)^{\beta_\delta}$$

$$f_i(\psi) = \rho_{0i} \left(\frac{\psi}{\psi_0}\right)^{p_i} \left(1-\frac{\psi^{s_i}}{\psi_0^{s_i}}\right)^{q_i}$$

the anisotropy profile is a strong generalization of the special cases mentioned before
(constant anisotropy, Osipkov-Merritt,…)

Baes & Van Hese 2007, A&A, 471, 419

# Analytical models

- a general seven-parameter family of augmented densities

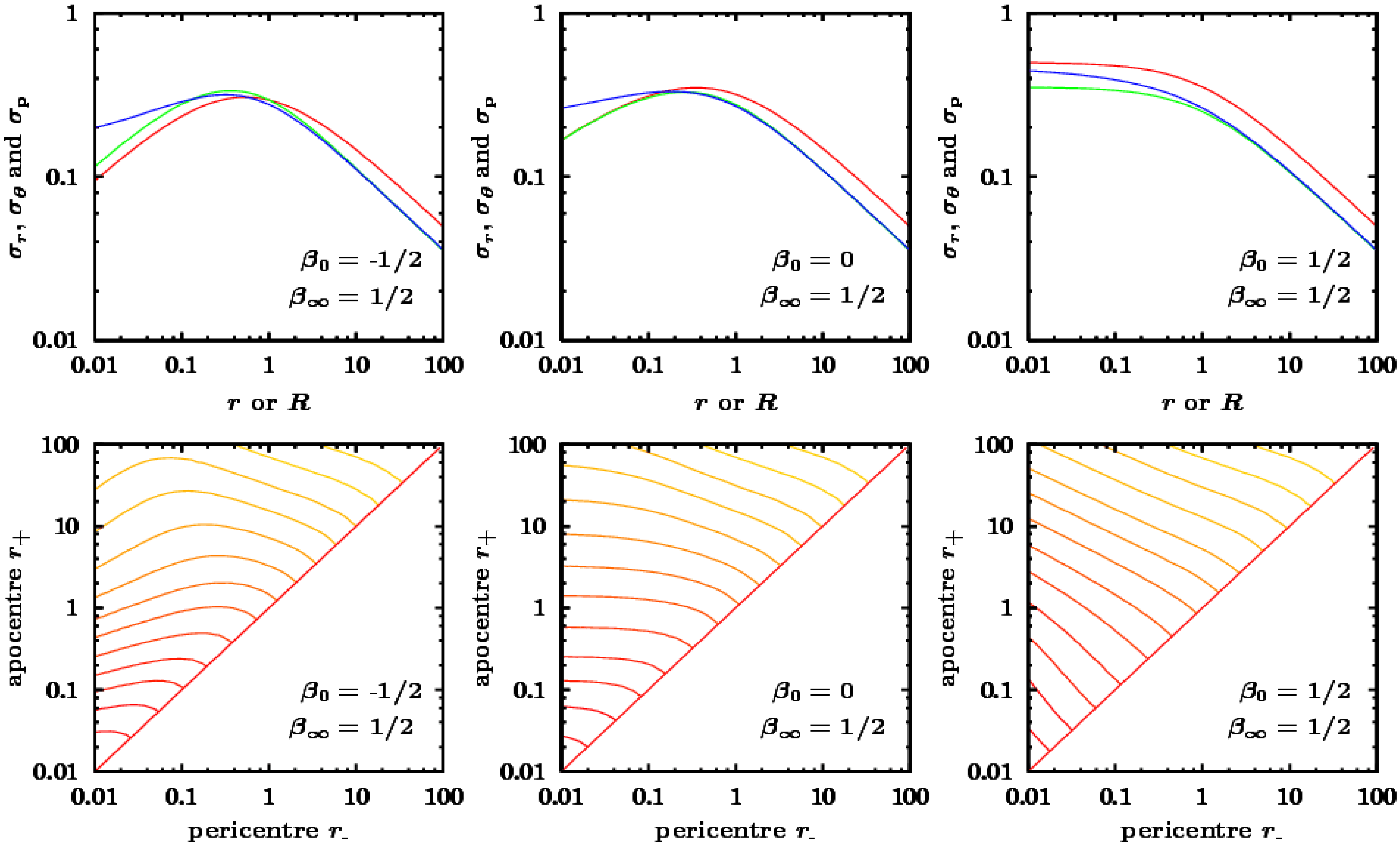
$$\tilde{\rho}_i(\psi, r) = \rho_{0i} \left(\frac{\psi}{\psi_0}\right)^{p_i} \left(1 - \frac{\psi^{s_i}}{\psi_0^{s_i}}\right)^{q_i} \left(\frac{r}{r_\mathrm{a}}\right)^{-2\beta_0} \left(1 + \frac{r^{2\delta}}{r_\mathrm{a}^{2\delta}}\right)^{\beta_\delta}$$

$$F_i(E, L) = \frac{\rho_{0i}}{M_\mathrm{tot}(2\pi\psi_0)^{3/2}} \sum_{j=0}^{\infty} (-1)^j \binom{q_i}{j} \left(\frac{E}{\psi_0}\right)^{p_i + j s_i - 3/2} \times \sum_{k=0}^{\infty} \binom{\beta_\delta}{k} \frac{\Gamma(1 + p_i + j s_i)}{\Gamma\left(p_i + j s_i - \frac{1}{2} + \beta_k\right)\Gamma\left(1 - \beta_k\right)} \left(\frac{L^2}{2 r_\mathrm{a}^2 E}\right)^{-\beta_k}$$

- contains (a.o.) self-consistent Plummer and Hernquist models
  - with arbitrary anisotropy at small and large radii
  - completely analytical dispersions, DF,…

Baes & Van Hese 2007, A&A, 471, 419

# Analytical Hernquist models




Baes & Van Hese 2007, A&A, 471, 419

how to generate arbitrary densities?

# More general analytical models

Quadratic Programming (Dejonghe 1989):

- construct a library of different components with the same anisotropy

$$\tilde{\rho}_i(\psi, r) = f_i(\psi) \left(\frac{r}{r_\mathrm{a}}\right)^{-2\beta_0} \left(1 + \frac{r^{2\delta}}{r_\mathrm{a}^{2\delta}}\right)^{\beta_\delta}$$

- create a set of $M$ density data points $\rho_\mathrm{obs}(r_m)$
- calculate the corresponding $\rho_i(r_m) = \tilde{\rho}_i\left(\psi(r_m), r_m\right)$

- select the best-fitting linear combination of $N$ components to the density data

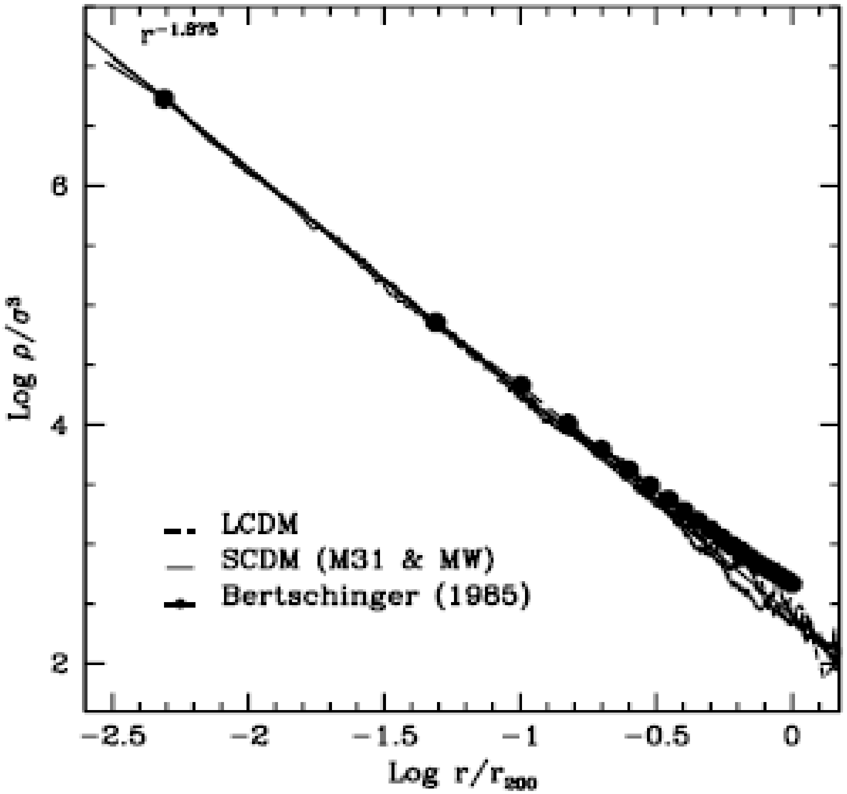
$$\chi_N^2 = \frac{1}{M} \sum_{m=1}^{M} w_m \left(\rho_\mathrm{obs}(r_m) - \sum_{i=1}^{N} a_{N,i}\, \rho_i(r_m)\right)^2$$

- the corresponding DF is then simply

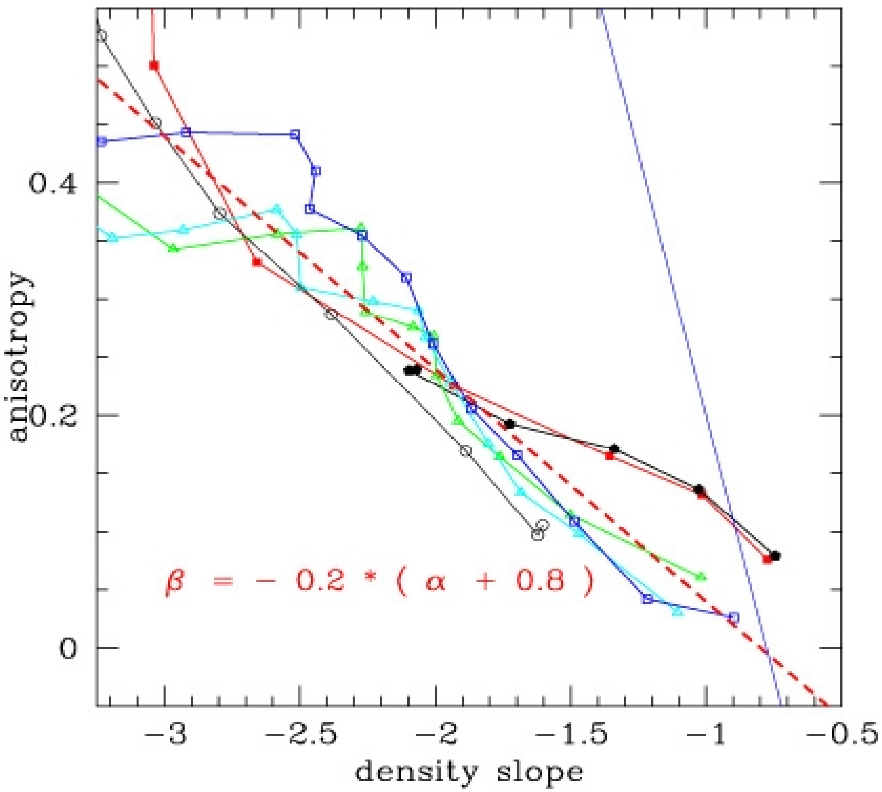
$$F(E, L) = \sum_{i=1}^{N} a_{N,i} F_i(E, L)$$

# Empirical properties of dark halos

- typically cusped in the central regions
- power-law behavior for "pseudo" phase-space density
- linear relationship between density slope and anisotropy




Taylor & Navarro 2001



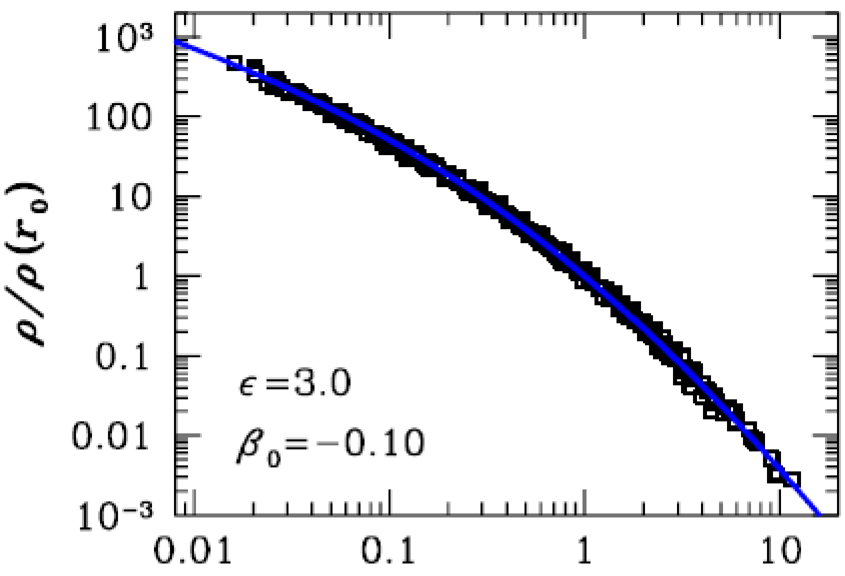

Hansen & Moore 2006

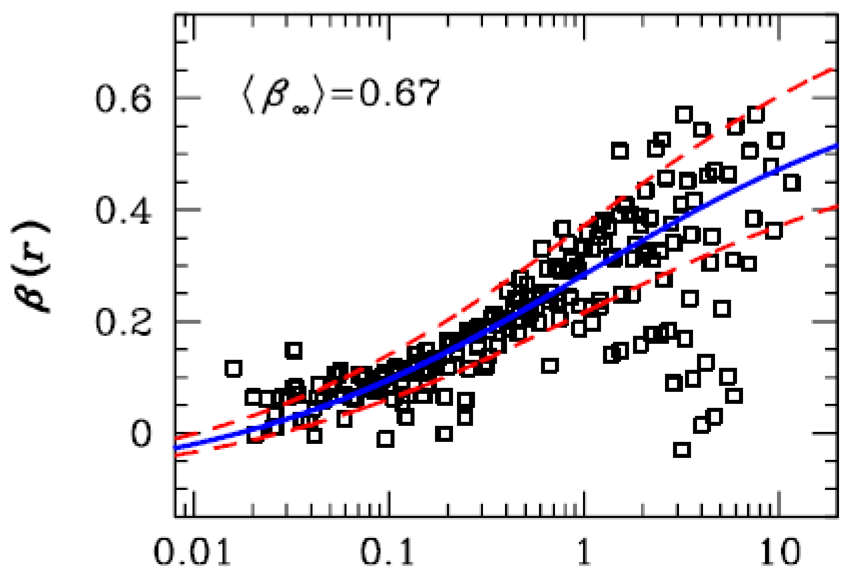
# Dark matter halo models

Dehnen & MacLaughlin (2005): Jeans models for dark halos based on these assumptions

- potential, density and dispersion are analytical
- anisotropy profile as in our components
- excellent fit to a set of simulated halos

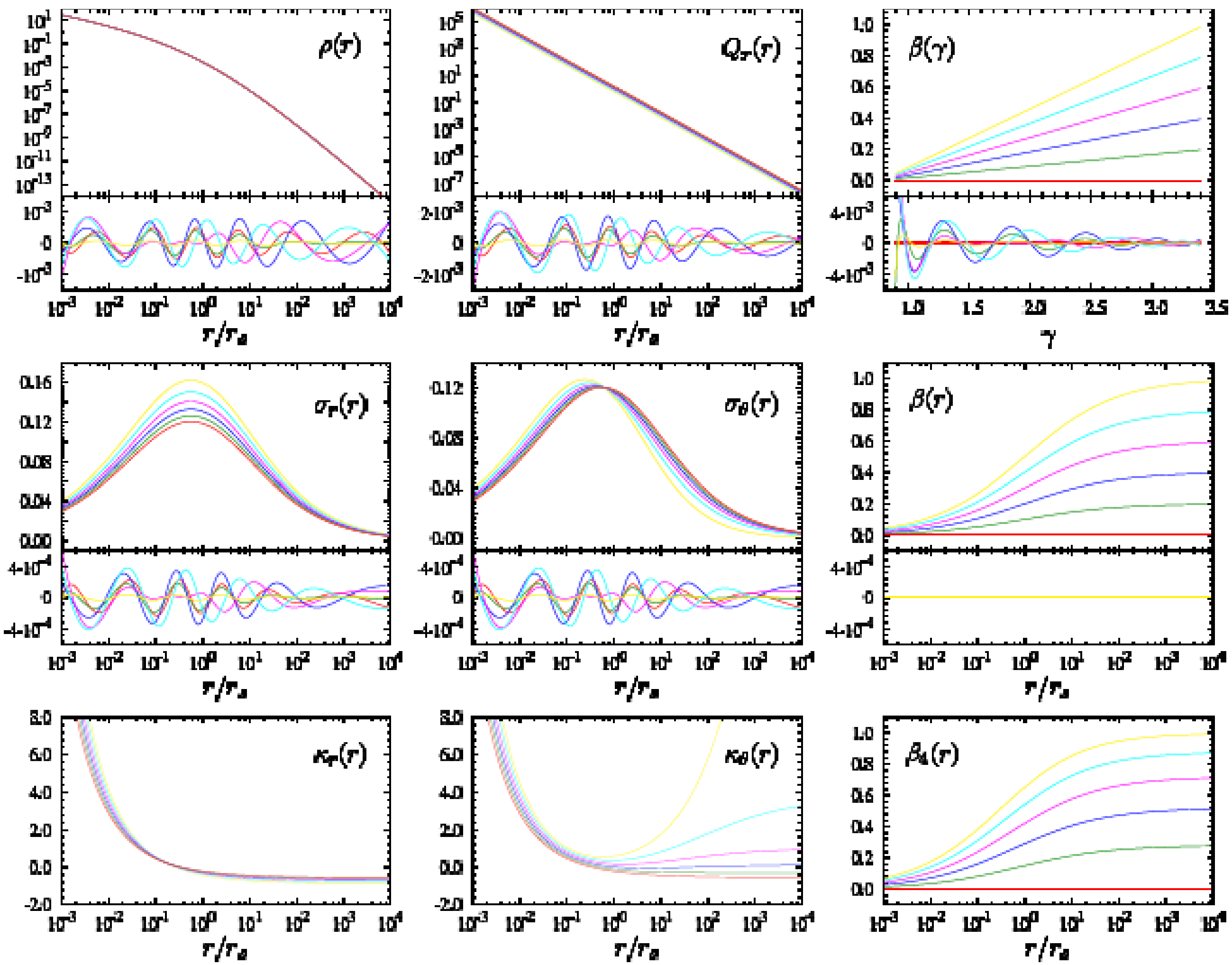
$$\gamma(r) = \frac{\gamma_0 + \gamma_\infty x^\eta}{1 + x^\eta} \qquad \frac{\rho}{\sigma_r^\varepsilon}(r) = \frac{\rho}{\sigma_r^\varepsilon}(r_s) \left(\frac{r}{r_s}\right)^{-\alpha_{crit}} \qquad \beta(r) = \frac{\beta_0 + \beta_\infty x^\eta}{1 + x^\eta}$$



# Dark matter halo models

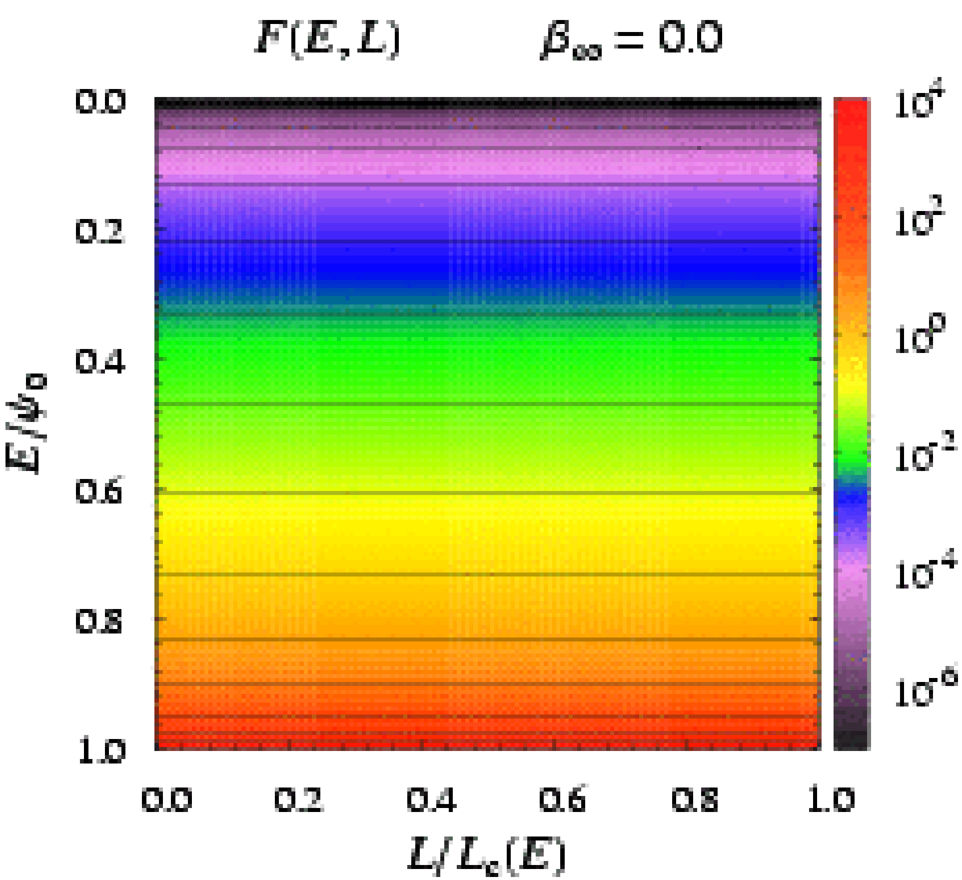
- dynamical models with $N$=10 components
- models with a range of outer anisotropies from 0 to 1
- quality of the fits is excellent

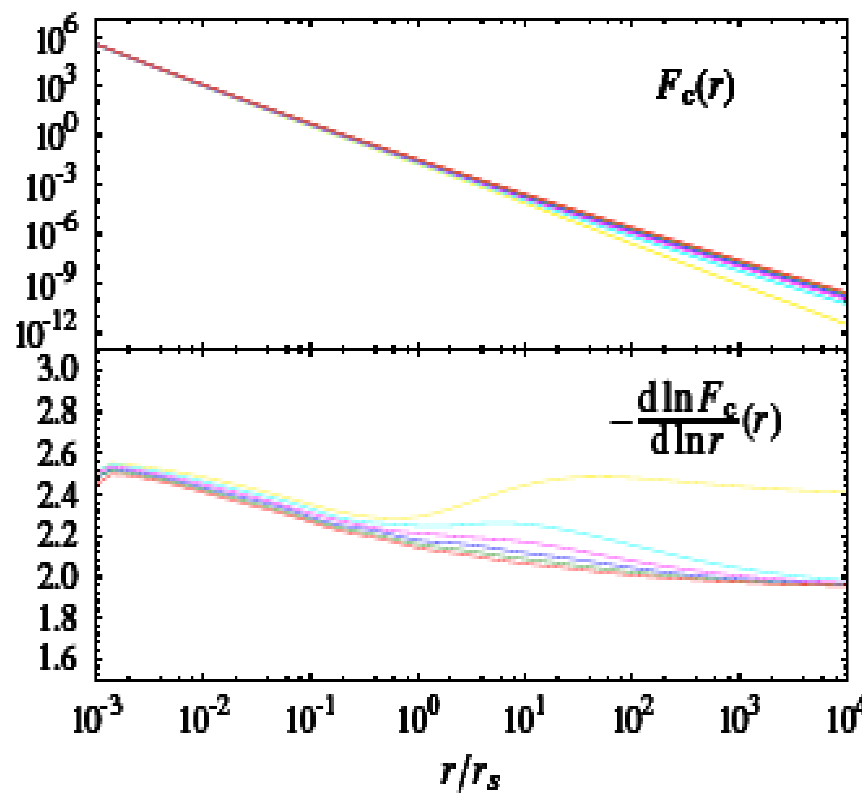



Van Hese, Baes & Dejonghe 2008, ApJ

# Distribution function

- DF positive for all anisotropies
- smoothly varying for $\beta_\infty$ ranging from 0 to 1
- remarkable: DF for circular orbits is power-law

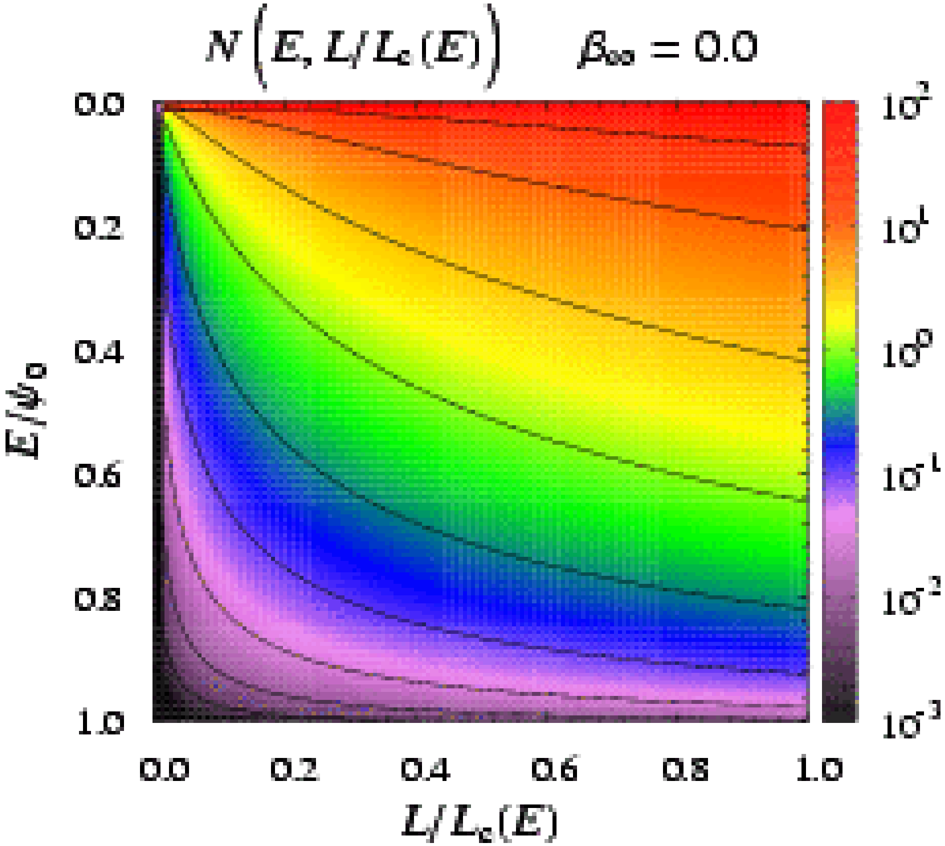


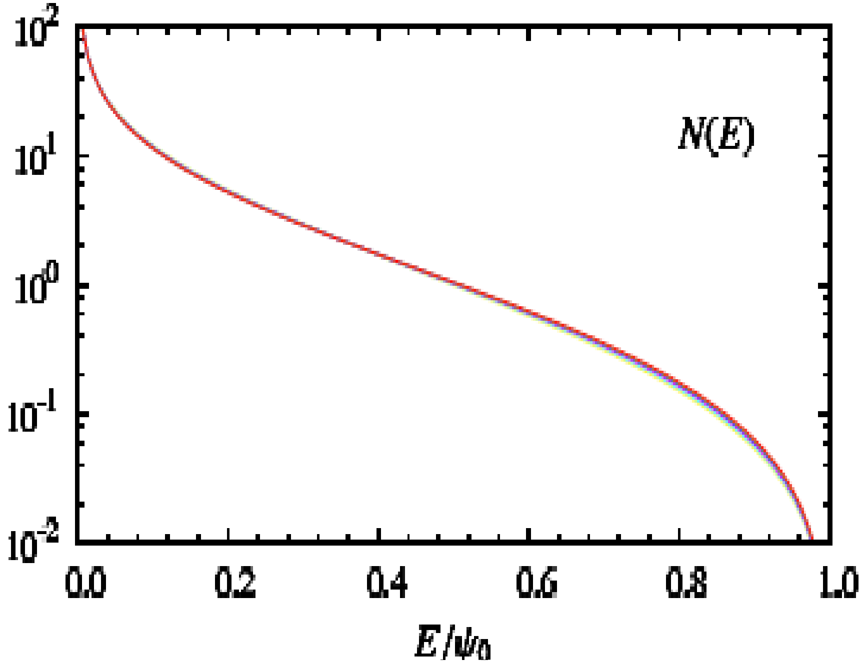




Van Hese, Baes & Dejonghe 2008, ApJ

# Differential distribution

- *N(E,L)* gives the number of orbits with integrals *E* and *L*
- *N(E)* nearly independent of the anisotropy



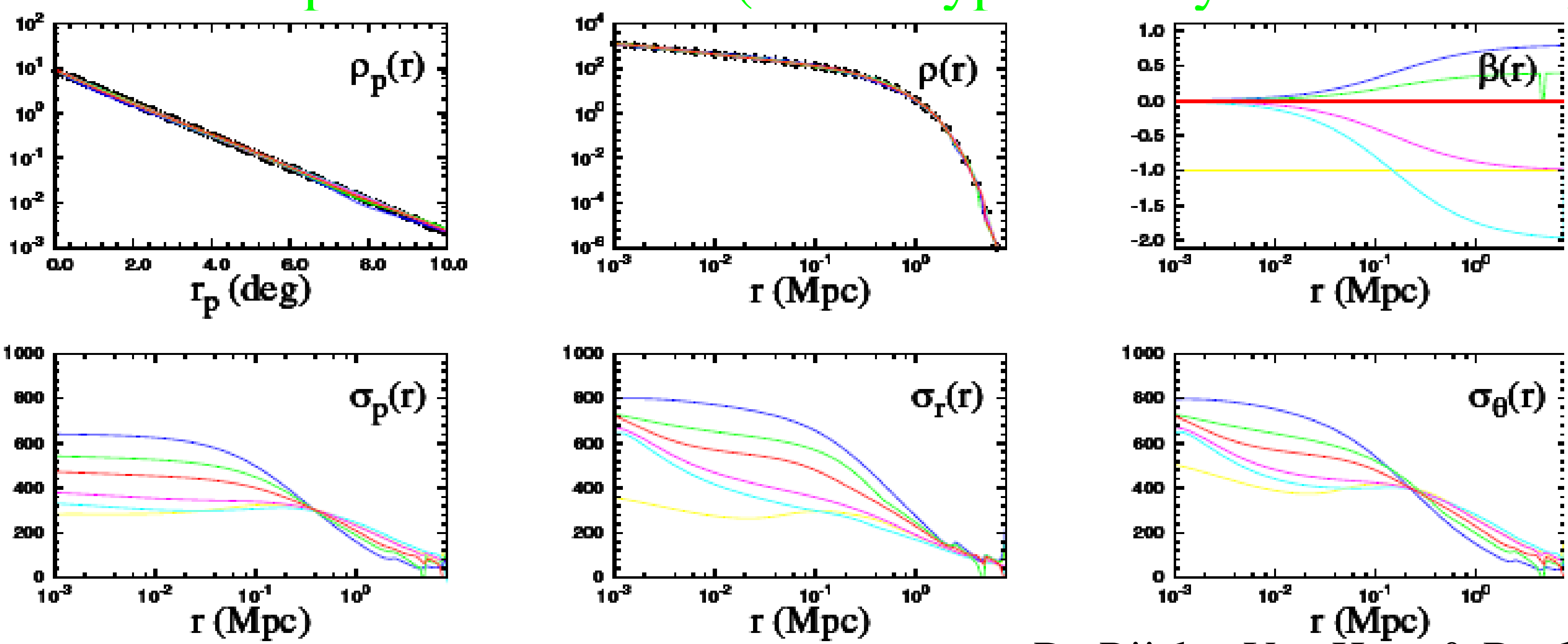




Van Hese, Baes & Dejonghe 2008, ApJ

# Future work

- Modeling different potentials and densities
- Gain more insight into the dynamical structure
- Check other “universal” properties (e.g. velocity distribution)
- Fitting directly to *N*-body data
- Modeling dark matter and galaxy distribution in clusters
- Extension to non-spherical halos

Dwarf Ellipticals in Fornax (Sersic-type density in NFW-halo)




De Rijcke, Van Hese & Buyle 2008

# Conclusions

- new technique to construct analytical dynamical models with arbitrary density and realistic anistropy profile
- useful in other dynamical studies, with other data moments than the density
- our models can generate initial conditions for *N*-body simulations
- phase-space investigation of dark matter halos with universal properties
  - very accurate fits
  - Jeans models of DM05 have positive DF
  - *N(E)* nearly independent of anisotropy

**valuable complementary approach to *N*-body studies**