Gravitational waves and lensing in the metric $f(R)$

theory proposed by Sobouti

SERGIO MENDOZA <sergio@mendoza.org>
http://www.mendoza.org/sergio

INSTITUTO DE ASTRONOMÍA
UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO (UNAM)

Talk available at
http://www.mendoza.org/sergio/talks

DM, DE & Alternative Gravity

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1 Overview of this talk

☆ Introduction: dark matter and modified theories of gravity.
☆ Gravitational waves in metric theories of gravity.
☆ Sobouti’s metric theory of gravity
☆ Gravitational lensing, Einstein’s deflection angle

References:
2 Introduction

☆ Astronomers use the word *dark* referring to any entity they cannot comprehend, e.g. dark matter and dark energy.

☆ Neptune was a dark matter candidate on its time! Bouvard (1821), Adams (1843) & Airy (also Le Verrier (1846) independently) postulated its existence because of the anomalous movement of Uranus.

☆ Le Verrier (1900’s) proposed that Mercury’s precession of its perihelium was caused by an unknown planet: *Vulcan*. It was never found.

☆ Einstein (1915) showed with its new relativistic theory of gravity that this precession can be accounted by modifications to Newton’s gravity.
3 Is gravitation valid at all physical scales?

★ General relativity tests have been made to physical scales less than the solar system size ($10^{11}$ m). At larger scales (galactic or cosmological) we either have to include Dark Matter and Dark Energy or to change our laws and/or models.

★ Is it possible that at greater orders of magnitude, say $10^{17}$ m (galactic sizes) standard general relativity needs modifications?
4 Dark matter

Flat rotation curves on spiral galaxies imply: missing matter (dark, cold non-baryonic) or modifications to gravity (MOND).
5 Accelerated expansion of the universe!

This type of expansion is usually attributed to scalar fields (cosmological constant, quintessence). However, one can modify gravity in order to react as a repulsive (rather than attractive) force (e.g. Capozziello et al., 2005, 2006). Note: Starobinsky (1980) modified gravity in order to account for an exponential growth of the scale factor in cosmology (like inflation).
6 Modified Newtonian Dynamics (MOND)

★ Milgrom (1983) proposed that in order to get flat rotation curves on spiral galaxies, Newton’s law of gravitation must be changed to:

\[
\frac{a^2}{a_0} = -G \frac{M}{r^2}.
\] (1)

with \(a_0 \approx 10^{-8} \text{ cm/s}^2 \approx cH_0\).

★ MOND was refined with the creation of AQUAL (Aquadratic Lagrangian), given by:

\[
L = -\frac{a_0^2}{8\pi G} \Lambda \left( \frac{|\nabla \phi|^2}{a_0^2} \right) - \rho \phi,
\] (2)

with \(-\nabla \phi = a\) and

\[
\Lambda(y) = \begin{cases} y, & \text{Newton } (a \gtrsim a_0) \\ y^{3/2}, & \text{MOND } (a \lesssim a_0). \end{cases}
\] (3)
7 Tensor Vector Scalar (TeVeS) - Bekenstein (2004)

- Problem with MOND is that it is not relativistic.
- There was an attempt to build a relativistic version of AQUAL (Bekenstein & Milgrom 1984), but gravitational waves were acausal and it does not reproduce gravitational lenses more than general relativity.
- TeVeS was the first attempt to build a relativistic theory of MOND.
- Can account for different phenomenology.
- Waves are subluminal and can account for additional bending to gravitational lensing.
- General problem: too complicated!!!
8 Metric f(R) theories of gravity

The Hilbert action is given by:

\[ S = -m \int ds - \frac{1}{2} \int f(R) \sqrt{-g} \, d^4x + \int L_m \sqrt{-g} \, d^4x. \]  

Variations of (4) produce

\[ \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0, \quad G_{\mu\nu} = T^{(\text{mat})}_{\mu\nu} + T^{(\text{curv})}_{\mu\nu}, \]  

where \( T^{(\text{mat})}_{\mu\nu} := T^{(\text{std-matt})}_{\mu\nu} / f'(R) \) and \( T^{(\text{curv})}_{\mu\nu} \) depends on \( f(R) \) and of its derivatives up to 4th order.
9 Gravitational waves in $f(R)$ theories

If one wants to think that a metric theory of gravity can account for different galactic phenomena, then it must not be acausal.

☆ Intuitively, one expects that gravitational waves travel at the speed of light $c = 1$ on a metric theory.

☆ In fact, we have shown (Mendoza & Rosas-Guevara 2007 and Jaime & Mendoza 2008) that this is only possible if and only if $f(R) = R^n \forall \text{ real } n \neq 0$.

☆ This is made by linearising field equations

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}.$$  \hspace{1cm} (6)

⇒ obtain a wave equation for $\Psi^{\mu\nu} := h^{\mu\nu} - \eta^{\mu\nu} \eta^{\alpha\beta} h_{\alpha \beta}/2$, with a propagation velocity equals to $c = 1$, in the Lorenz gauge.

☆ Note: it can also be done for a perturbation of the form

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}.$$  \hspace{1cm} (7)
10 Which function $f(R)$ reproduces the observed phenomenology in galaxies?

★★ At the time of Sobouti’s proposal, Mendoza & Rosas–Guevara were trying to match $f(R) = R^{3/2}$. This can be shown not to work (e.g. it doesn’t flatten rotation curves).

★★ A few months after Sobouti’s theory, Capozziello (2006) proposed $f(R) = R^{(1.34-2.41)}$, by solving the 4th order differential equations and doing statistics on a group of spiral galaxies.

★★ Sobouti’s trick: choose $f(R)$ for a Schwarzschild-like metric in order that the parametric quantity $F(r, \alpha) := df/dR$ is s.t. $F(r, \alpha) \to 1$ as $\alpha \to 0$, i.e. $\alpha \ll 1$.

★★ The metric coefficients of the Schwarzschild-like metric can thus be obtained and the resulting equations are of 2nd order (Sobouti, 2007).
With this it is found that \( f \) (Sobouti, 2007) is given by

\[
f(R) = (3\alpha)^{\alpha/2} R^{(1-\alpha/2)} \approx R \left[ 1 - \frac{\alpha}{2} \ln R + \frac{\alpha}{2} \ln(3\alpha) \right].
\]  

(8)

11 Motion of a particle on the field

At first order of approximation, a star is treated as a test particle orbiting about a central object.

Orbit is obtained from geodesic equation.

In this way it is possible to obtain the circular velocity of the star:

\[
v^2 \approx \frac{1}{2} \alpha c^2 + \frac{GM}{r} \left[ 1 + \frac{1}{2} \alpha \left\{ -1 + \ln \left( \frac{rc^2}{2GM} \right) \right\} \right].
\]  

(9)

The velocity tends to an asymptotic value \( v_\infty \) as a test particle goes away from the mass distribution and \( \alpha = 2v_\infty/c^2 \).
The relation between $\alpha$ and the mass $M$ is not given. However, Sobouti made a general statistical study using 40 spiral galaxies and assuming that the velocity is proportional to $M^{1/4}$, in order to satisfy the Tully–Fisher relation. With this, it follows that:

$$\alpha = \left(2.8 \times 10^{-12}\right) \left(M/M_\odot\right)^{1/2}$$  \hspace{1cm} (10)

Note that this is not the best way to fit the parameter $\alpha$, since the distribution of matter on a spiral galaxy is not spherically symmetric. Using gravitational lenses from cluster of galaxies Bernal & Mendoza (2008) have obtained a more reliable value of $\alpha = 3.5 \times 10^{-9} \left(M/M_\odot\right)^{1/2}$. 
12 Light deflection for a system with spherical symmetry

The deflection of light that passes near a compact object is calculated from a null geodesic \((ds = 0)\).

The net deflection angle \(\beta\) is found to be

\[
\beta = \pi \left[ \frac{2\sqrt{1 - \alpha}}{2 - \alpha} - 1 \right] + 2\sqrt{1 - \alpha} \left( \frac{r_S}{r_m} \right)^{1-\alpha/2}.
\]

\[(11)\]
The fluctuation $\Delta \beta / \beta_E$ measures the deviations this angle has from general relativity:

$$\frac{\Delta \beta}{\beta_E} = \frac{\beta}{\beta_E} - \frac{\beta_E}{\beta_E}.$$  \hfill (12)
13 Conclusions

- The metric $f(R)$ theory proposed by Sobouti (2007) is in principle a good candidate for a relativistic modified theory of gravity that seems to account for different observed phenomena.
- Gravitational waves on a metric theory $f(R) \propto R^n$ -which includes Sobouti’s one- are luminal.
- It’s possible to obtain more bending from this $f(R)$ theory, as compared to standard general relativity.
- We’re doing more analysis for gravitational waves, and also, working more on astrophysical aspects of gravitational lenses.
14 Generic problem with metric theories applied to galactic phenomena

Soussa (2003) made a strong argument saying that a metric theory of gravitation can reproduce MOND and gravitational lenses phenomenology if certain conditions on the stability of the theory are violated.

We have shown that Sobouti’s metric theory of gravity does not agree with the theorem.