Gravitational waves and lensing in the metric $f(R)$ theory proposed by Sobouti

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DM, DE & Alternative Gravity

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1 Overview of this talk

- ☆ Introduction: dark matter and modified theories of gravity.
- ☆ Gravitational waves in metric theories of gravity.
- ☆ Sobouti's metric theory of gravity
- ☆ Gravitational lensing, Einstein's deflection angle

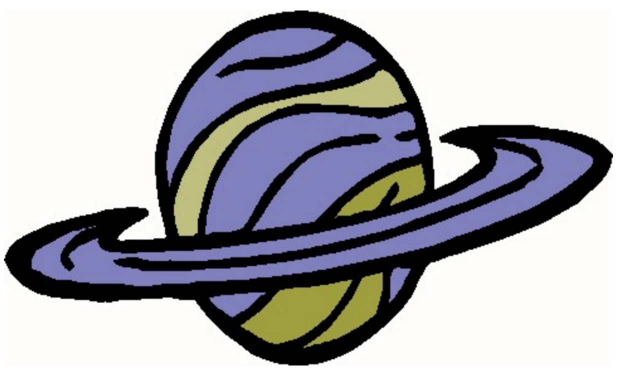
References:

Sobouti (2007), Capozziello (2007), Mendoza & Rosas-Guevara (2007).

BSc thesis of Yetli M. Rosas–Guevara (available at http://bc.unam.mx).

2 Introduction

- ☆ Astronomers use the word *dark* referring to any entity they cannot comprehend, e.g. dark matter and dark energy.
- ☆ Neptune was a dark matter candidate on its time! Bouvard (1821), Adams (1843) & Airy (also Le Verrier (1846) independently) postulated its existence because of the anomalous movement of Uranus.
- ☆ Le Verrier (1900's) proposed that Mercury's precession of its perihelium was caused by an unknown planet: *Vulcan*. It was never found.
- ☆ Einstein (1915) showed with its new relativistic theory of gravity that this precession can be accounted by modifications to Newton's gravity.

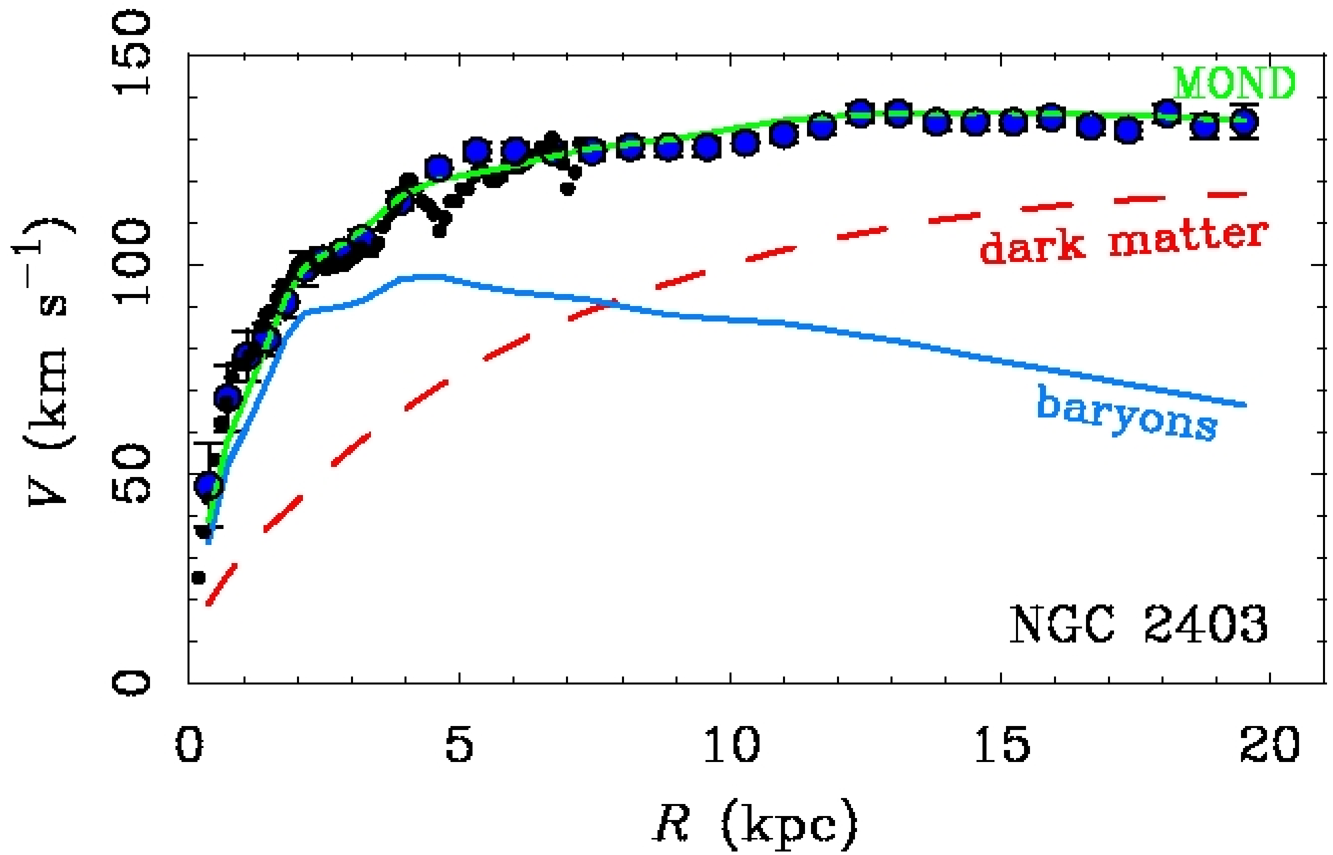


Dark matter at the beginning of the 20th century

3 Is gravitation valid at all physical scales?

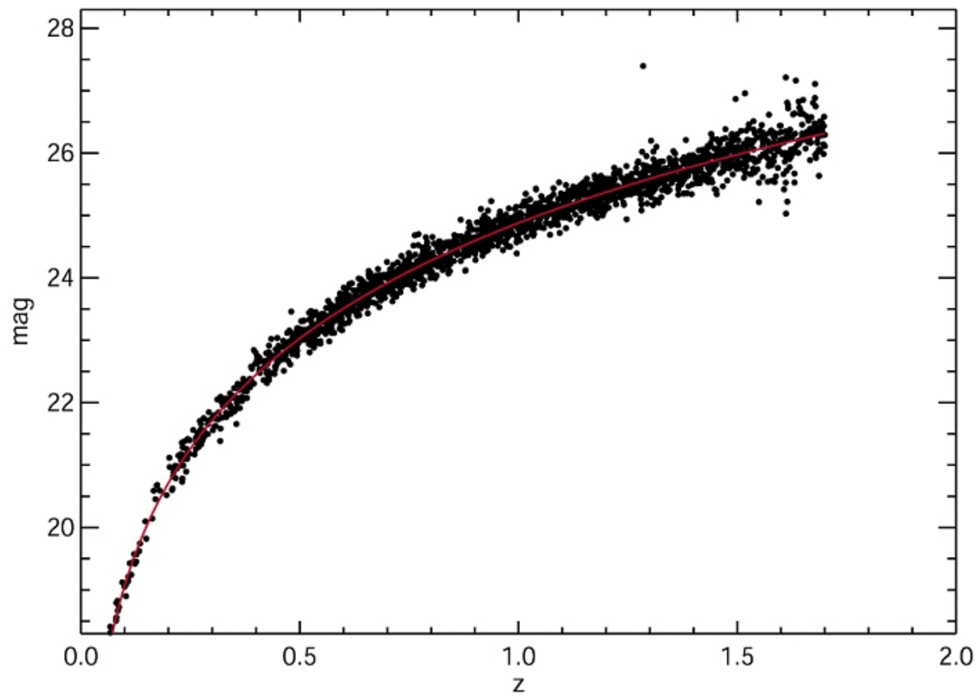
- ☆ General relativity tests have been made to physical scales less than the solar system size (10^{11}m). At larger scales (galactic or cosmological) we either have to include Dark Matter and Dark Energy or to change our laws and/or models.
- ☆ Is it possible that at greater orders of magnitude, say 10^{17}m (galactic sizes) standard general relativity needs modifications?

4 Dark matter



Flat rotation curves on spiral galaxies imply: missing matter (dark, cold non-baryonic) or modifications to gravity (MOND).

5 Accelerated expansion of the universe!



This type of expansion is usually attributed to scalar fields (cosmological constant, quintessence). However, one can modify gravity in order to react as a repulsive (rather than attractive) force (e.g. Capozziello et al., 2005, 2006). Note: Starobinsky (1980) modified gravity in order to account for an exponential growth of the scale factor in cosmology (like inflation).

6 Modified Newtonian Dynamics (MOND)

- ☆ Milgrom (1983) proposed that in order to get flat rotation curves on spiral galaxies, Newton's law of gravitation must be changed to:

$$\frac{\boldsymbol{a}^2}{a_0} = -G\frac{M}{r^2}. \tag{1}$$

with $a_0 \approx 10^{-8}\,\mathrm{cm/s^2} \approx cH_0$.

- ☆ MOND was refined with the creation of AQUAL (Aquadratic Lagrangian), given by:

$$L = -\frac{a_0^2}{8\pi G}\Lambda\left(\frac{|\nabla\phi|^2}{a_0^2}\right) - \rho\phi, \tag{2}$$

with $-\nabla\phi = \boldsymbol{a}$ and

$$\Lambda(y) = \begin{cases} y, & \text{Newton} \quad (a \gtrsim a_0) \\ y^{3/2}, & \text{MOND } (a \lesssim a_0). \end{cases} \tag{3}$$

7 Tensor Vector Scalar (TeVeS) -Bekenstein (2004)

- ☆ Problem with MOND is that it is not relativistic.
- ☆ There was an attempt to build a relativistic version of AQUAL (Bekenstein & Milgrom 1984), but gravitational waves were acausal and it does not reproduce gravitational lenses more than general relativity.
- ☆ TeVeS was the first attempt to build a relativistic theory of MOND.
- ☆ Can account for different phenomenology.
- ☆ Waves are subluminal and can account for additional bending to gravitational lensing.
- ☆ General problem: too complicated!!!

8 Metric f(R) theories of gravity

☆ The Hilbert action is given by:

$$S = -m \int \mathrm{d}s - \frac{1}{2} \int f(R)\, \sqrt{-g}\, \mathrm{d}^4 x + \int L_{\mathrm{m}} \sqrt{-g}\, \mathrm{d}^4 x. \tag{4}$$

☆ Variations of (4) produce

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau} \frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0, \qquad G_{\mu\nu} = T^{(\mathrm{mat})}_{\mu\nu} + T^{(\mathrm{curv})}_{\mu\nu}, \tag{5}$$

where $T^{(\mathrm{matt})}_{\mu\nu} := T^{(\mathrm{std\text{-}matt})}_{\mu\nu} / f'(R)$ and $T^{(\mathrm{curv})}_{\mu\nu}$ depends on $f(R)$ and of its derivatives up to 4th order.

9 Gravitational waves in $f(R)$ theories

If one wants to think that a metric theory of gravity can account for different galactic phenomena, then it must not be acausal.

- ✰ Intuitively, one expects that gravitational waves travel at the speed of light $c = 1$ on a metric theory.
- ✰ In fact, we have shown (Mendoza & Rosas-Guevara 2007 and Jaime & Mendoza 2008) that this is only possible if and only if $f(R) = R^n$ $\forall$ real $n \neq 0$.
- ✰ This is made by linearising field equations

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{6}$$

$\Rightarrow$ obtain a wave equation for $\Psi^{\mu\nu} := h^{\mu\nu} - \eta^{\mu\nu} h^{\alpha}{}_{\alpha}/2$, with a propagation velocity equals to $c = 1$, in the Lorenz gauge.

- ✰ Note: it can also be done for a perturbation of the form

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}. \tag{7}$$

10 Which function $f(R)$ reproduces the observed phenomenology in galaxies?

- ☆ At the time of Sobouti's proposal, Mendoza & Rosas–Guevara were trying to match $f(R) = R^{3/2}$. This can be shown not to work (e.g. it doesn't flatten rotation curves).
- ☆ A few months after Sobouti's theory, Capozziello (2006) proposed $f(R) = R^{(1.34–2.41)}$, by solving the 4th order differential equations and doing statistics on a group of spiral galaxies.
- ☆ Sobouti's trick: choose f(R) for a Schwarzschild-like metric in order that the parametric quantity $F(r, \alpha) := \mathrm{d}f/\mathrm{d}R$ is s.t. $F(r, \alpha) \to 1$ as $\alpha \to 0$, i.e. $\alpha \ll 1$.
- ☆ The metric coefficients of the Schwarzschild-like metric can thus be obtained and the resulting equations are of 2nd order (Sobouti, 2007).

☆ With this it is found that f (Sobouti, 2007) is given by

$$f(R) = (3\alpha)^{\alpha/2} R^{(1-\alpha/2)} \approx R[1 - \frac{\alpha}{2} \ln R + \frac{\alpha}{2} \ln(3\alpha)]. \tag{8}$$

11 Motion of a particle on the field

☆ At first order of approximation, a star is treated as a test particle orbiting about a central object.

☆ Orbit is obtained from geodesic equation.

☆ In this way it is possible to obtain the circular velocity of the star:

$$v^2 \approx \frac{1}{2}\alpha c^2 + \frac{GM}{r} \left[1 + \frac{1}{2}\alpha \left\{ -1 + \ln\left(\frac{rc^2}{2GM} \right) \right\} \right]. \tag{9}$$

☆ The velocity tends to an asymptotic value v_∞ as a test particle goes away from the mass distribution and $\alpha = 2v_\infty/c^2$.

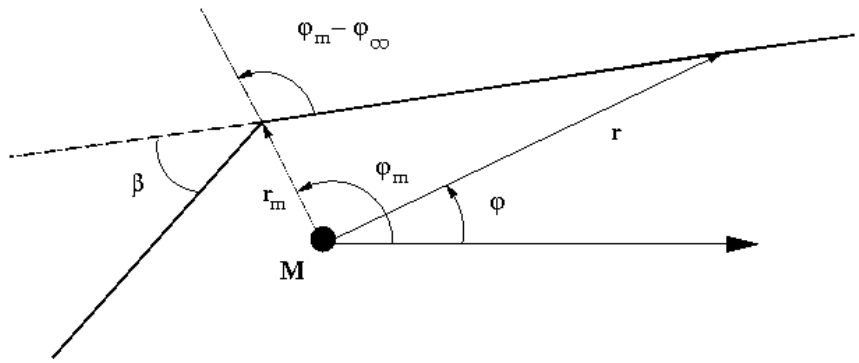
☆ The relation between α and the mass M is not given. However, Sobouti made a general statistical study using 40 spiral galaxies and assumming that the velocity is proportional to $M^{1/4}$, in order to satisfy the Tully–Fisher relation. With this, it follows that:

$$\alpha = \left(2.8 \times 10^{-12}\right) (M/M_{\odot})^{1/2} \tag{10}$$

☆ Note that this is not the best way to fit the parameter α, since the distribution of matter on a spiral galaxy is not spherically symmetric. Using gravitational lenses from cluster of galaxies Bernal & Mendoza (2008) have obtained a more reliable value of $\alpha = 3.5 \times 10^{-9} (M/M_{\odot})^{1/2}$.

12 Light deflection for a system with spherical symmetry

☆ The deflection of light that passes near a compact object is calculated from a null geodesic ($\mathrm{d}s = 0$).

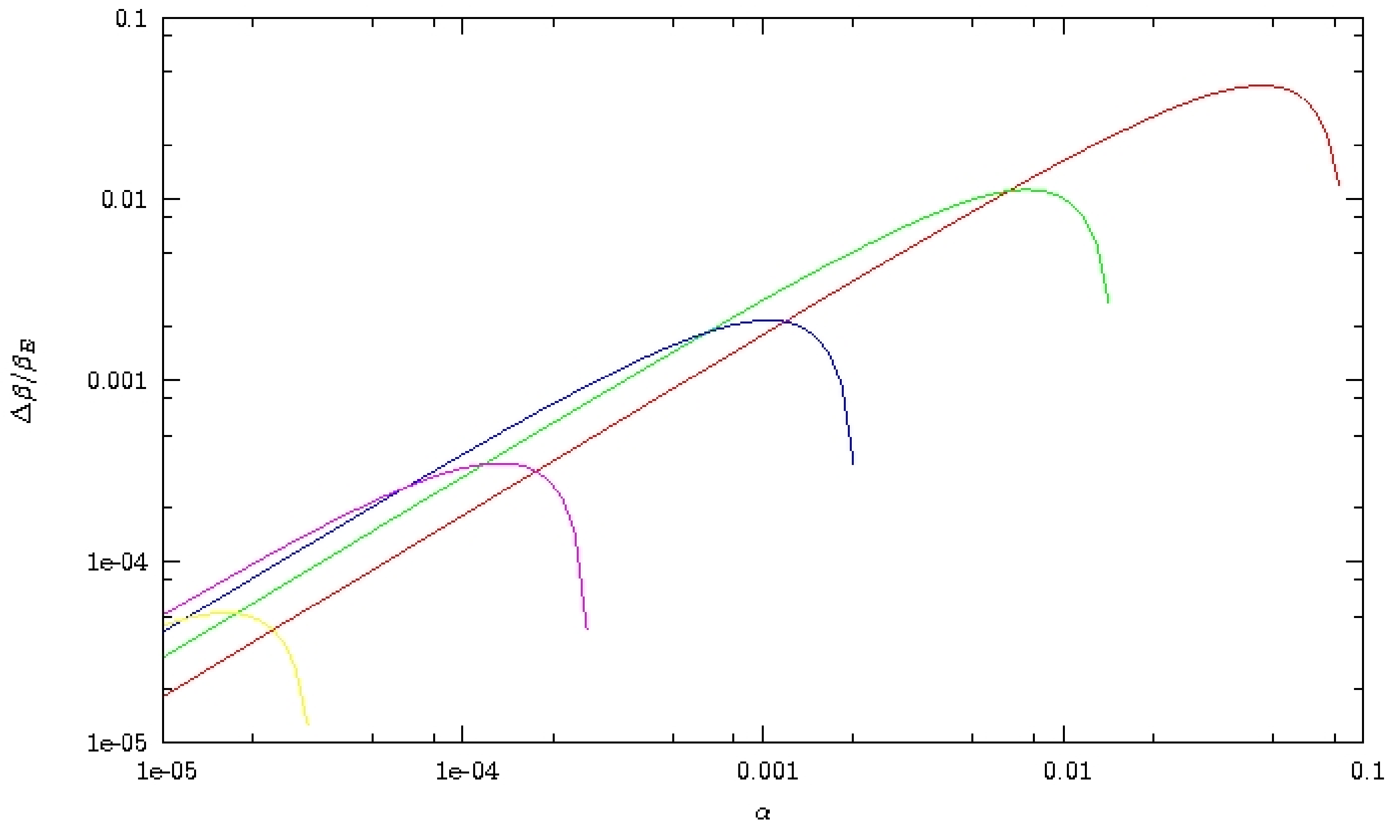


☆ The net deflection angle β is found to be

$$\beta = \pi \left[\frac{2\sqrt{1-\alpha}}{2-\alpha} - 1 \right] + 2\sqrt{1-\alpha} \left(\frac{r_\mathrm{S}}{r_m} \right)^{1-\alpha/2} . \qquad (11)$$

☆ The fluctuation $\Delta\beta/\beta_{\mathrm{E}}$ measures the deviations this angle has from general relativity:

$$\frac{\Delta\beta}{\beta_{\mathrm{E}}} = \frac{\beta - \beta_{\mathrm{E}}}{\beta_{\mathrm{E}}}. \tag{12}$$



13 Conclusions

- ☆ The metric $f(R)$ theory proposed by Sobouti (2007) is in principle a good candidate for a relativistic modified theory of gravity that seems to account for different observed phenomena.
- ☆ Gravitational waves on a metric theory $f(R) \propto R^n$ -which includes Sobouti's one- are luminal.
- ☆ It's possible to obtain more bending from this $f(R)$ theory, as compared to standard general relativity.
- ☆ We're doing more analysis for gravitational waves, and also, working more on astrophysical aspects of gravitational lenses.

14 Generic problem with metric theories applied to galactic phenomena

- ☆ Soussa (2003) made a strong argument saying that a metric theory of gravitation can reproduce MOND and gravitational lenses phenomenology if certain conditions on the stability of the theory are violated.
- ☆ We have shown that Sobouti's metric theory of gravity does not agree with the theorem.