A Markov Chain Monte Carlo for Galactic Cosmic Ray Physics

I. Method and results for the Leaky Box Model

[Putze et al., arXiv:0808.2437v1]

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Motivation

1) The propagation parameters are linked to/necessary for

- transport in the turbulent magnetic fields
- cosmic-ray source properties (spectral indices, source abundances)
- γ-ray diffuse emission, indirect dark matter detection ($\bar{p}$, $\bar{d}$, $e^+, \ldots$)

2) GeV data for nuclei are being taken (last time was HEAO-3 in 1979)

- Satellite: PAMELA (see e.g. this conference)
- Balloons: ATIC, CREAM, TRACER, ...

$\Rightarrow$ *Requires a sound numerical tool*

The MCMC is widely used in cosmology, why not for CRs?

- Waterproof (robust, statistically sound, efficient...)
- Handy (tackles large parameter space, chains run in parallel)

Extracting the target function $P(\boldsymbol{\theta}|\mathrm{data})$: principle

Model depending on m parameters: $\boldsymbol{\theta} = \{\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(m)}\}$

$\Downarrow$

Bayes' theorem [for the likelihood, we use $\mathcal{L}(\boldsymbol{\theta}) = \exp\left(-\frac{\chi^2(\boldsymbol{\theta})}{2}\right)$]

$$\underbrace{P(\boldsymbol{\theta}|\mathrm{data})}_{\text{posterior probability}} \propto \underbrace{P(\mathrm{data}|\boldsymbol{\theta})}_{\text{likelihood}} \cdot \underbrace{P(\boldsymbol{\theta})}_{\text{prior probability}}$$

$\Downarrow$

MCMC algorithms tell you how to explore the parameter space

$P(\boldsymbol{\theta}|data)$ is sampled simply by generating a chain of n points:

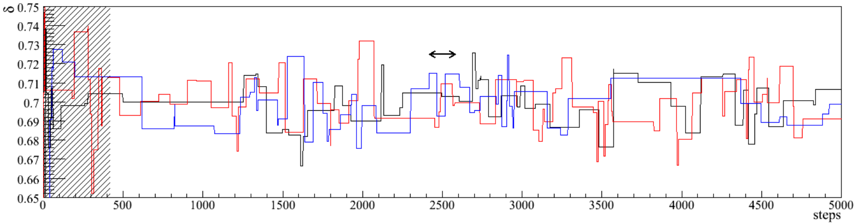
$$\{\boldsymbol{\theta}_i\}_{i=1,\ldots,n} \equiv \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_n\}$$

$\Downarrow$

Marginalisation to get $P(\theta^i|\mathrm{data})$

Practical example: chain analysis

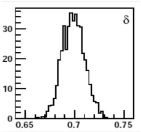
3 chains (black, blue and red) of 5000 steps each: only δ is shown



- Discard burn-in length (hatched area)
- Keep only $1/l$ point, where l is the correlation length (arrow)

$\Rightarrow$ stack values of the chain in an histogram (marginalising is easy!)

$\Rightarrow$ PDF of the parameter



$\Rightarrow$

- Extract mean value
- Extract confidence intervals/levels

$$\mathrm{CL}(x) \equiv \int_{\Delta_x} \mathcal{P}(\theta_i)\mathrm{d}\theta_i = 1 - x$$

Technical details: the Metropolis-Hastings algorithm

Markov Chain: *what is a jump?*

The transition probability $\mathcal{T}$ to jump from $\boldsymbol{\theta}_i$ to $\boldsymbol{\theta}_{i+1}$ only depends on $\boldsymbol{\theta}_i$

$$\Rightarrow \mathcal{T}(\boldsymbol{\theta}_{i+1}|\boldsymbol{\theta}_1,\ldots,\boldsymbol{\theta}_i) = \mathcal{T}(\boldsymbol{\theta}_{i+1}|\boldsymbol{\theta}_i)$$

Proposal density (trial function): *how to jump?*

From the current position $\boldsymbol{\theta}_i$, a trial state $\boldsymbol{\theta}_{\rm trial}$ is formed from the proposal density q

$$\Rightarrow \text{generate } \boldsymbol{\theta}_{\rm trial} \text{ from } q(\boldsymbol{\theta}_{\rm trial}|\boldsymbol{\theta}_i)$$

Metropolis-Hastings: *do we accept all jumps?*

The MH algorithm ensures that the chain tends asymptotically to the target PDF, if the jump is accepted with a certain probability: $[\mathcal{T}(\boldsymbol{\theta}_{i+1}|\boldsymbol{\theta}_i) = a_{\rm MH}(\boldsymbol{\theta}_{\rm trial}|\boldsymbol{\theta}_i) q(\boldsymbol{\theta}_{\rm trial}|\boldsymbol{\theta}_i)]$

$$a_{\rm MH}(\boldsymbol{\theta}_{\rm trial}|\boldsymbol{\theta}_i) \equiv \min\left(1, \frac{P(\boldsymbol{\theta}_{\rm trial}|data)}{P(\boldsymbol{\theta}_i|data)} \frac{q(\boldsymbol{\theta}_i|\boldsymbol{\theta}_{\rm trial})}{q(\boldsymbol{\theta}_{\rm trial}|\boldsymbol{\theta}_i)}\right)$$

$\Rightarrow$ if accepted, add in the chain $\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_{\rm trial}$

$\Rightarrow$ if rejected, add in the chain $\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i$

Model and parameterizations

1) LBM with minimal reacceleration (Seo & Ptuskin, 1994)

- Contains most of CR phenomenology
- Simple, so adapted for a first study

$$\lambda_{\rm esc}(R) = \begin{cases} \lambda_0 \beta {R_0}^{-\delta} & \text{when } R < R_0, \\ \lambda_0 \beta R^{-\delta} & \text{otherwise;} \end{cases}$$

$$K_{pp} = \frac{4}{3} {\mathcal{V}_a}^2 \frac{\tau_{\rm esc}}{\delta(4-\delta^2)(4-\delta)}$$

2) Source: shape and normalization

- Power law at high energy, modified at low energy
- Abundances q_j rescaled to (or extracted from) data

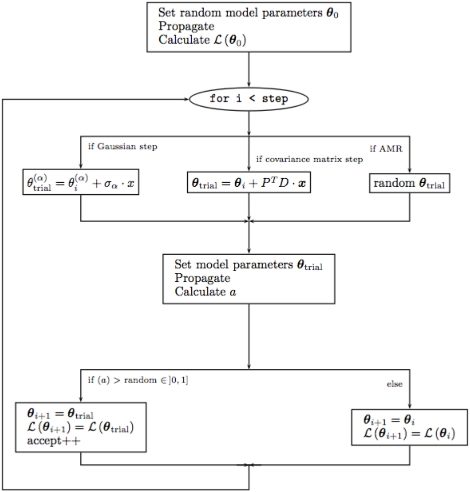
$$Q(E) = q_j \beta^{\eta_j} R^{-\alpha_j}$$

$$\alpha_j \equiv \alpha \quad \text{and} \quad \eta_j \equiv \eta \; (= -1)$$

3) Data

- B/C data (HEAO-3) + low energy (ACE, Voyager...): pdf of $\{\lambda_0,\ R_0,\ \delta,\ \mathcal{V}_a\}$
- O data from HEAO-3 and TRACER: pdf of $\{\alpha,\ \eta\}$
- CNO from HEAO-3: pdf of $\{q_C,\ q_N,\ q_O\}$

Summary: flow chart and inputs



MCMC related inputs

- Number of parameters
- Trial function
- Number and length of chains

Propagation/source related inputs

- Transport and source parameters
- Propagation equation & solution

User input (this study)

- Propagation model: LBM
- Free parameters: $\{\lambda_0, R_0, \delta, \mathcal{V}_a\}$, but also $\{\alpha, \eta, q_C, q_N, q_O\}$
- Data used: B/C, CNO

Sequential use of three trial functions

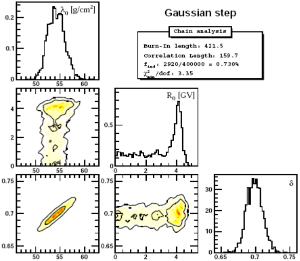
1) Gaussian step
⇒ gross determination

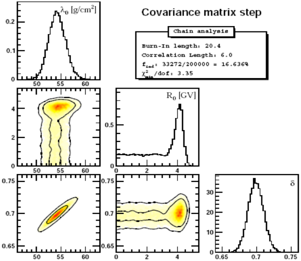
2) Covariance Matrix
⇒ takes care of correlations

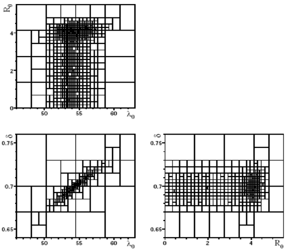
3) Binary Space Partitioning
⇒ fine determination

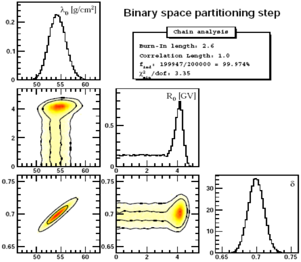
⇒ each method provides an input to the next

[coarse to fine description]





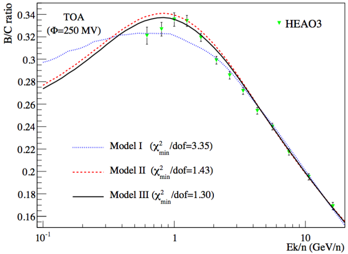


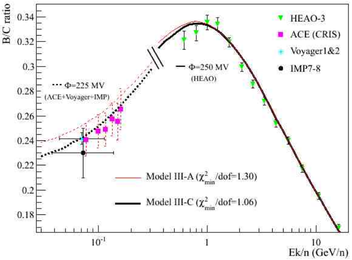


Fitting transport parameters with B/C data

Model	λ_0 g cm^{-2}	R_0 GV	δ	$\mathcal{V}_a$ km s^{-1}kpc^{-1}	$\chi^2_{\rm min}$/dof
I-A	54^{+2}_{-2}	$4.2^{+0.3}_{-0.9}$	$0.70^{+0.01}_{-0.01}$	-	3.35
II-A	26^{+2}_{-2}	-	$0.52^{+0.02}_{-0.02}$	88^{+6}_{-11}	1.43
III-A	30^{+5}_{-4}	$2.8^{+0.6}_{-0.8}$	$0.58^{+0.01}_{-0.06}$	75^{+10}_{-13}	1.30
III-C	27^{+2}_{-2}	$2.6^{+0.4}_{-0.7}$	$0.53^{+0.02}_{-0.03}$	86^{+9}_{-5}	1.06

Testing different models (I, II and III) or different data sets:
A = HEAO-3 data alone (14 data points), C = HEAO-3 + ACE + Voyager1&2 + IMP7-8 (22 data points)





⇒ Model I compatible with results of Webber et al. (ApJ 508, 1998)
⇒ Model III in agreement with diffusion model from Maurin et al. (A&A 294, 2002) for reacceleration
⇒ best model requires R_0 and $\mathcal{V}_a$

HEAO-3+low energy: confidence region and envelopes

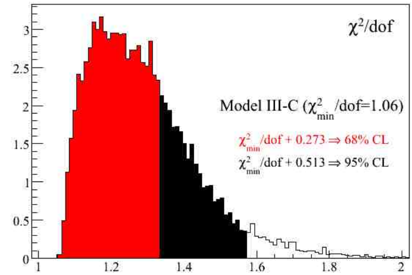
Independently of the statistical meaning of a model, the confidence interval can be extracted from the cumulative χ^2 PDF:

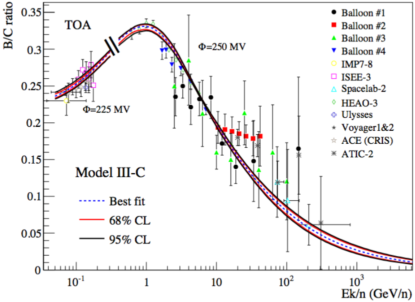
$$\int_{\chi^2_{\rm min}}^{\chi^2_{\rm min}+\Delta\chi^2} \mathcal{P}(\chi^2)\mathrm{d}\chi^2 = 1-\gamma$$

- Best fit parameters provide the most likely CR fluxes
- 1D marginalised PDF gives the most likely value of the parameter

$\Rightarrow$ both do not always coincide

Confidence levels on fluxes (B/C, $\bar{\rm p}$, ...) must be constructed from a sampling of the (still) correlated parameters...





Fitting simultaneously B/C and O

⇒ Adjusting simultaneously $\lambda_{\rm esc}$ and $Q(E) = q_j \beta^{\eta} R^{-\alpha}$ for O

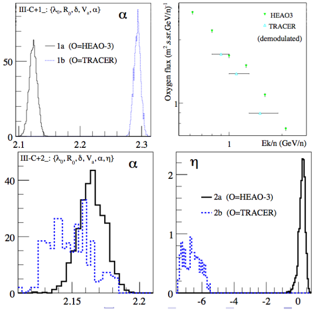
Model	λ_0 g cm^{-2}	R_0 GV	δ	$\mathcal{V}_a$ km s^{-1}kpc^{-1}	α	η
III+1a	37^{+2}_{-2}	$4.4^{+0.1}_{-0.2}$	$0.61^{+0.01}_{-0.01}$	64^{+4}_{-4}	$2.124^{+0.005}_{-0.007}$	-
III+1b	$20.9^{+0.2}_{-0.8}$	$0.3^{+0.6}_{-0.1}$	$0.47^{+0.01}_{-0.01}$	103^{+2}_{-3}	$2.294^{+0.004}_{-0.006}$	-
III+2a	29^{+2}_{-2}	$2.7^{+0.3}_{-0.4}$	$0.55^{+0.01}_{-0.02}$	84^{+4}_{-7}	$2.16^{+0.01}_{-0.01}$	$0.3^{+0.1}_{-0.2}$
III+2b	32^{+4}_{-1}	$4.3^{+0.3}_{-0.1}$	$0.56^{+0.03}_{-0.01}$	62^{+2}_{-2}	$2.14^{+0.03}_{-0.01}$	$-6.7^{+0.9}_{-0.1}$

- Model 1: α only is free ($\eta = -1$)
- Model 2: α and η are free

[Dataset a/b: HEAO-3/TRACER (Ave et al., 2008) for O flux]

⇒ η absorbs uncertainties from either the modulation or the low energy spectral shape

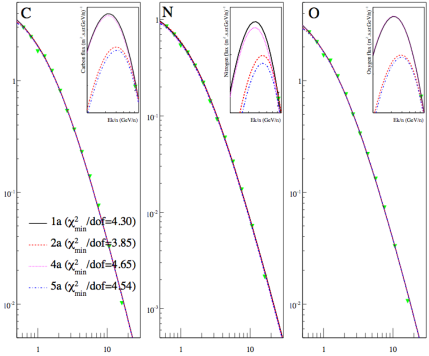
⇒ "Unbiased" determination of $\alpha \approx 2.15$ for HEAO-3 or TRACER (in the LBM)



Fitting simultaneously B/C and CNO

⇒ Adjusting simultaneously $\lambda_{\rm esc}$ and $Q(E) = q_j \beta^\eta R^{-\alpha}$ for CNO HEAO-3 data

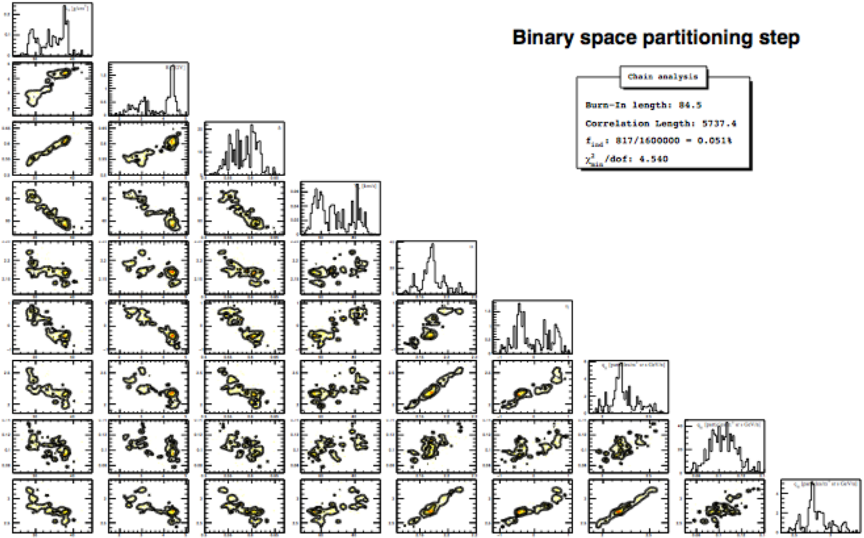
Model	α	η	$10^{20} \times (q_C \vert q_N \vert q_O)$ $(\mathrm{m}^3\ \mathrm{s\ GeV/n})^{-1}$
III+4	$2.13^{+0.01}_{-0.01}$	-	$1.93^{+0.04}_{-0.004} \vert 0.089^{+0.007}_{-0.005} \vert 2.42^{+0.04}_{-0.05}$
III+5	$2.17^{+0.02}_{-0.02}$	$-0.4^{+1.2}_{-0.1}$	$2.2^{+0.2}_{-0.1} \vert 0.107^{+0.01}_{-0.006} \vert 2.7^{+0.3}_{-0.1}$



⇒ relative CNO abundances OK with Engelmann et al. 1990 (A&A 233)

⇒ source spectrum parameters are correlated to propagation parameters

Correlations between propagation and source parameters



Binary space partitioning step

Chain analysis

Burn-In length: 84.5

Correlation Length: 5737.4

f_{ind}: 817/1600000 = 0.051%

χ^2_{min}/dof: 4.540

Conclusions

MCMC algorithm:

- Successful implementation of the MCMC to extract posterior PDF for GCRs
- Sequential use of three different trial functions (zoning for a fine description)
- Numerous chains run in parallel (quick and efficient)
- Full coverage of large parameter space (up to 9 in this study)

Results on CR physics:

- Kolmogorov spectral index of 1/3 excluded (as found in Maurin et al., 2001)
- Model with R_0 and $\mathcal{V}_a$ preferred (aka wind and reacc. in diffusion models)
- Confidence interval on δ for the best model: $\delta = 0.53^{+0.02}_{-0.03}$
- Correlations between transport source parameters: unbiased value for $\alpha \approx 2.15$
- ... and can help to diagnose inconsistent sets of data (helpful for spectra)

Processing soon...

$\Rightarrow$ take advantage of new published data
$\Rightarrow$ use a more up-to-date model (diffusion model)
$\Rightarrow$ extend the analysis on larger data sets (more nuclear species)

`--- Other things than can be further studied ---`

- **Anti-protons:** new PAMELA data up to 100 GeV, could be used for a cross-check of the secondary origin of this flux (related to dark matter indirect searches)
- **Source slopes:** check the hypothesis $\alpha_i \neq \alpha_j$ for different species and/or diagnose some problems in the data if we believe the slope should be universal
- **Low energy spectra:** take advantage of H and He species to characterize the IS low energy spectra
- Source abundances, fit of e^-, e^+ and γ fluxes, ...

$\Rightarrow$ First public release of the propagation code USINE soon...
$\Rightarrow$ MCMC package released later (next year?)...